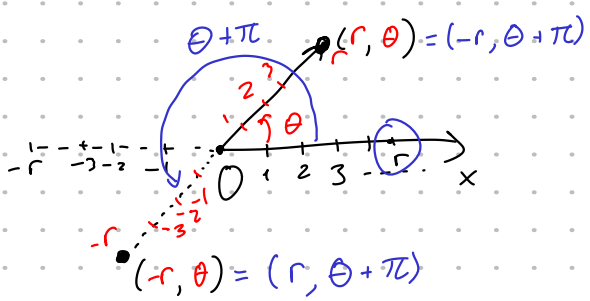
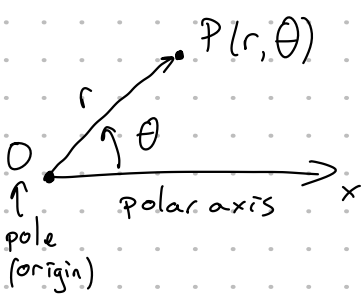
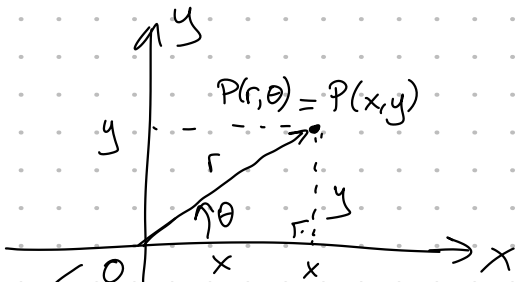
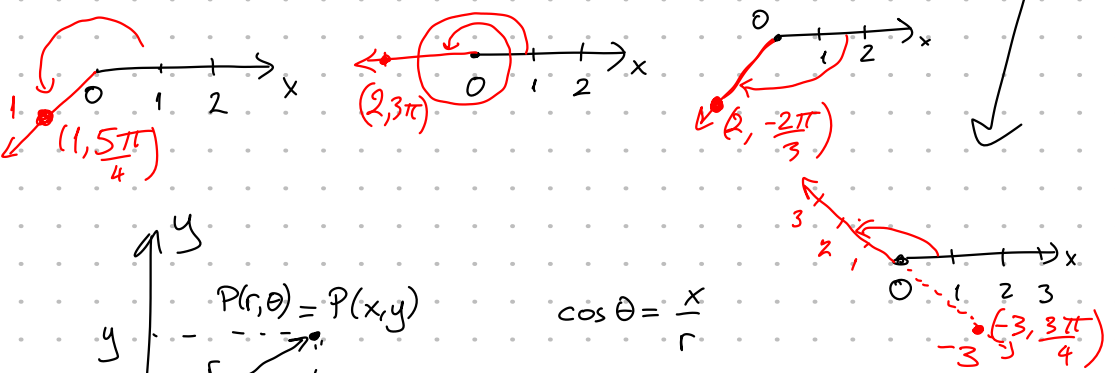


10.3 Polar Coordinates



Example Plot the points whose polar coordinates are given.

- a) $(1, \frac{5\pi}{4})$ b) $(2, 3\pi)$ c) $(2, -\frac{2\pi}{3})$ d) $(-3, \frac{3\pi}{4})$



$$\cos \theta = \frac{x}{r}$$

$$\sin \theta = \frac{y}{r}$$

$$\Rightarrow x = r \cos \theta$$

$$y = r \sin \theta$$

$$r^2 = x^2 + y^2$$

$$\tan \theta = \frac{y}{x}$$

Example Convert the point $(2, \frac{\pi}{3})$ from polar to Cartesian coordinates.

$$r = 2, \theta = \frac{\pi}{3}, \quad x = r \cos \theta = 2 \cos \frac{\pi}{3} = 2 \left(\frac{1}{2}\right) = 1$$

$$y = r \sin \theta = 2 \sin \frac{\pi}{3} = 2 \left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$$

Example Represent the point with Cartesian coordinates $(1, -1)$ in terms of polar coordinates.

$$x = 1, y = -1$$

$$r^2 = x^2 + y^2 = 1 + 1 = 2$$

$$r^2 = 2$$

$$\tan \theta = \frac{y}{x} = \frac{-1}{1} = -1$$

$$\tan \theta = -1$$

$$r = \sqrt{2} \quad \theta = -\frac{\pi}{4}$$

$$r^2 = x^2 + y^2 = 1 + 1 = 2$$

$$r^2 = 2$$

$$\tan \theta = \frac{y}{x} = \frac{1}{-1} = -1$$

$$\tan \theta = -1$$

$$r = -\sqrt{2} \quad \theta = -\frac{\pi}{4}$$

$$\tan \theta = \frac{y}{x}$$

$$\hookrightarrow \theta = \arctan\left(\frac{y}{x}\right) \text{ then } -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

If $x > 0$ then r should be positive
If $x < 0$ then r should be negative

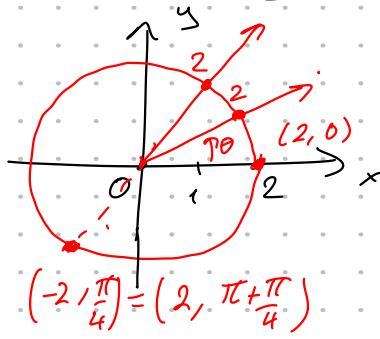
Polar Curves

$$r = f(\theta) \quad (\text{or more generally } F(r, \theta) = 0)$$

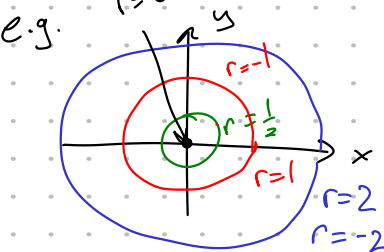
Example What curve is represented by the polar equation $r = 2$?

$$r = 2?$$

$$(r, \theta) = (2, \theta)$$



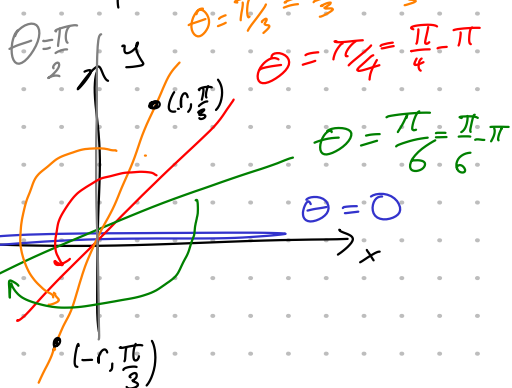
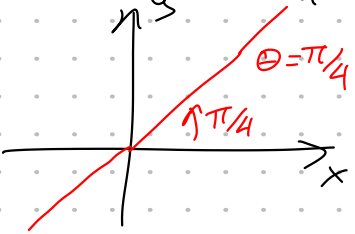
Circle!



$(r, \theta) = (0, \theta)$ ← origin for any θ

Example Sketch the polar curve $\theta = \frac{\pi}{4}$

$$(r, \theta) = (r, \frac{\pi}{4})$$

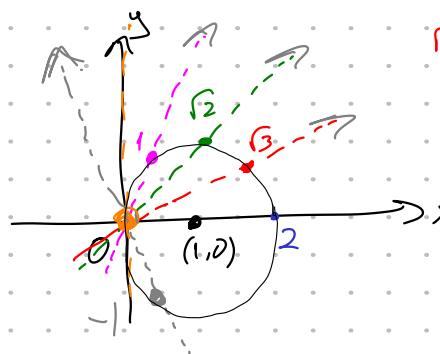


Example

a) Sketch the curve with polar equation $r = 2 \cos \theta$

b) Find a Cartesian equation for this curve.

θ	$r = 2 \cos \theta$
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
$\pi/2$	0
$2\pi/3$	-1



$$r = 2 \cos(\frac{5\pi}{4})$$

$$r = -\frac{\sqrt{2}}{2} = -\sqrt{2}$$

Circle!

$$r = 2 \cos \theta \rightarrow r^2 = 2 r \cos \theta$$

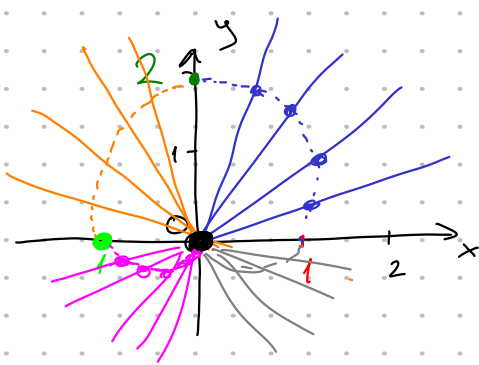
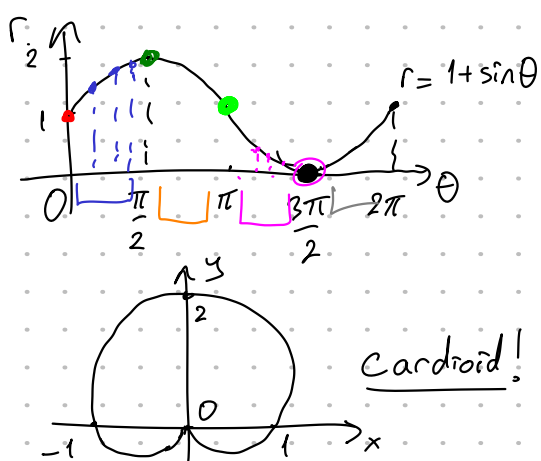
$$x^2 + y^2 = 2x$$

$$x^2 - 2x + 1 + y^2 = 1$$

$(x - 1)^2 + y^2 = 1$ circle of radius 1 centered at $(1, 0)$

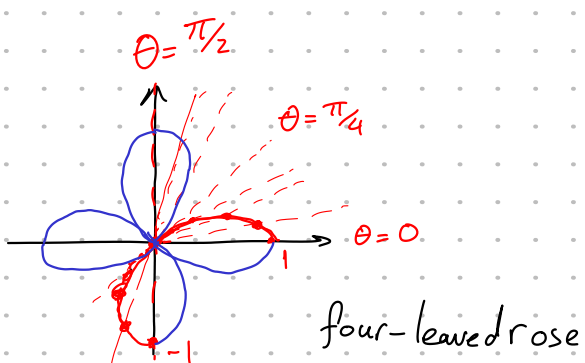
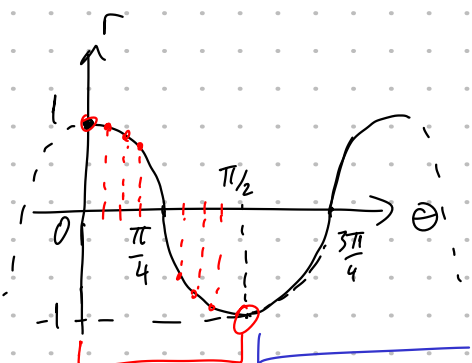
$$\begin{cases} r^2 = x^2 + y^2 \\ x = r \cos \theta \end{cases}$$

Example Sketch the curve $r = 1 + \sin\theta$



Cardioid!

Example Sketch the curve $r = \cos 2\theta$



four-leaved rose