Math 162 Calculus 2

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Office hours: Mon, Wed 3:30-5:00 pm (Starting this Wednesday)

Zoom link on Blackboard (same link as the lecture)

You can attend office hours of any other MTH 162 professor or TA.

Textbook: Calculus: Early Transcendentals (9th edition) by James Stewart

Grade:

- Webwork HW 15% Every week due on Friday 11:59 pm
- Workshop attendance and participation 10%
- 3 Midterm exams 25% each
- Final exam 25%

(Webcam Required during exams and workshops)

Lowest MT score will be dropped.
Lowest WW-HW will be dropped.
Lowest 2 Workshop scores will be dropped.
No WW-Quizzes this semester!

To access WW, you need to click on the link on Blackboard.
WW-Set 0 is a warmup set due this Friday, Feb 5.

Workshops will start next week. Signup will be available this Wed, Feb 3 at 6:30 pm.

Recall: FTC 1

If $g(x) = \int_a^x f(t)\,dt$ then $g'(x) = f(x)$

FTC 2

If $F' = f$ then $\int_a^b f(x)\,dx = F(b) - F(a)$

u-subs.

$$\int f(g(x))\,g'(x)\,dx = \int f(u)\,du$$

$u = g(x)$
$du = g'(x)\,dx$

Example If $F(x) = \int_3^x \frac{1}{1+t^3}\,dt$, $F'(x) = ?$

$$F'(x) = \frac{1}{1+x^3}$$

Example If $h(x) = \int_{-5}^{\sin x} (\cos(t^4) + t)\,dt$, $h'(x) = ?$

set $g(x) = \int_{-5}^{x} (\cos(t^4) + t)\,dt$ then $h(x) = g(\sin x)$

By the Chain Rule, $h'(x) = g'(\sin x)(\sin x)'$

By FTC 1, $g'(x) = \cos(x^4) + x$, thus

$$h'(x) = (\cos(\sin^4 x) + \sin x)\cos x$$

Example Given $f(x) = \begin{cases} -9x & \text{if } x \le 0 \\ \sin x & \text{if } x > 0, \end{cases}$

find $\int_{-\pi}^{\pi} f(x)\,dx$.

$$\int_{-\pi}^{\pi} f(x)\,dx = \int_{-\pi}^{0} f(x)\,dx + \int_{0}^{\pi} f(x)\,dx = -9\int_{-\pi}^{0} x\,dx + \int_0^{\pi} \sin x\,dx$$

$$= -9\frac{x^2}{2}\Big|_{-\pi}^{0} + (-\cos x)\Big|_0^{\pi} = 0 + \frac{9}{2}\pi^2 + \underbrace{(-\cos\pi)}_{+1} - \underbrace{(-\cos 0)}_{+1}$$

$\cos\pi = -1$
$\cos 0 = 1$

$$= 2 + \frac{9}{2}\pi^2$$

Example $\int_0^{\pi/2} \cos(x)\sin(\sin(x))\,dx$

$u = \sin x$
$du = \cos x\,dx$

$$= \int_0^1 \sin u\,du$$

$x = \pi/2 \Rightarrow u = \sin\pi/2 = 1$
$x = 0 \Rightarrow u = \sin 0 = 0$

$$= -\cos u\Big|_0^1 = -\cos 1 - (-\cos 0) = 1 - \cos 1$$

Example $\int \frac{1 - \sin^2 x}{\cos x}\,dx$ $(\cos^2 x + \sin^2 x = 1)$

$$= \int \frac{\cos^2 x}{\cos x}\,dx = \int \cos x\,dx = \sin x + C$$

Example $\int \frac{x-3}{(x^2-6x+3)^5}\,dx$

$u = x^2-6x+3$

$du = (2x-6)\,dx$

$du = 2(x-3)\,dx$

$\frac{1}{2}du = (x-3)\,dx$

$= \frac{1}{2}\int \frac{du}{u^5} = \frac{1}{2}\int u^{-5}\,du$

$= \frac{1}{2}\left(\frac{u^{-4}}{-4}\right) + C = -\frac{1}{8}(x^2-6x+3)^{-4} + C$

Example $\int \frac{3x}{1+x^4}\,dx$ let's try $u = 1+x^4$

$du = 4x^3\,dx$

$du = 4x^2 x\,dx$

Technically we can do

$u = 1+x^4 \qquad u-1 = x^4 \qquad \sqrt{u-1} = x^2$

then we have to deal with square roots and so on. There is an easier way!

$\int \frac{3x}{1+x^4}\,dx$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{3}{2}\int \frac{du}{1+u^2}$

$= \frac{3}{2}\arctan(u) + C = \frac{3}{2}\arctan(x^2) + C$

Example $\int x\sqrt[3]{x-1}\,dx$

$u = x-1$

$du = dx$

$= \int (u+1)\sqrt[3]{u}\,du$

$= \int (u+1)u^{1/3}\,du = \int (u^{4/3} + u^{1/3})\,du$

$= \frac{3}{7}u^{7/3} + \frac{3}{4}u^{4/3} + C = \frac{3}{7}(x-1)^{7/3} + \frac{3}{4}(x-1)^{4/3} + C$

Example $\int \frac{x^3}{\sqrt{x-1}}\,dx$

$u = x-1$

$du = dx$

$= \int \frac{(u+1)^3}{\sqrt{u}}\,du = \int (u+1)^3 u^{-1/2}\,du$

$= \int (u^3 + 3u^2 + 3u + 1)u^{-1/2}\,du = \int (u^{5/2} + 3u^{3/2} + 3u^{1/2} + u^{-1/2})\,du$

$= \frac{2}{7}u^{7/2} + 3\left(\frac{2u^{5/2}}{5}\right) + 3\left(\frac{2u^{3/2}}{3}\right) + 2u^{1/2} + C$

$= \frac{2}{7}(x-1)^{7/2} + \frac{6}{5}(x-1)^{5/2} + 2(x-1)^{3/2} + 2(x-1)^{1/2} + C$