6.4 Work

Mass : How much stuff there is? Independent of whether the object is in Earth's gravitational field or in the outer space

Weight: This depends on the gravitational field the object is in.

$$\text{density} = \frac{\text{mass}}{\text{unit "space"}} \ \text{(typically volume)}$$

Force: the ability to change an object's motion

Force = mass × acceleration

$$F = ma$$

The units

In the SI metric system	In the US Customary system
a: (m/s^2)	
m: (kg)	
F: $\left(N = kg\,\frac{m}{s^2}\right)$ (N: newtons)	F: (lb)

Work = Force × Distance (if the force is constant)

$(J = N \cdot m)$ (J: Joules)

the US system

Work = Force × distance

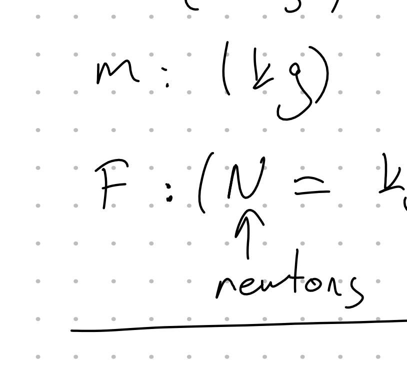
$(ft\text{-}lb) = (lb) \cdot (ft)$

$1\,ft\text{-}lb \simeq 1.36\,J$

Example a) How much work is done in lifting a 1.2 kg book off the floor to put it on a desk that is 0.7 m high? Use the fact that the acceleration due to gravity is $g = 9.8\ m/s^2$

b) How much work is done in lifting a 20-lb weight 6 ft off the ground?

(a)



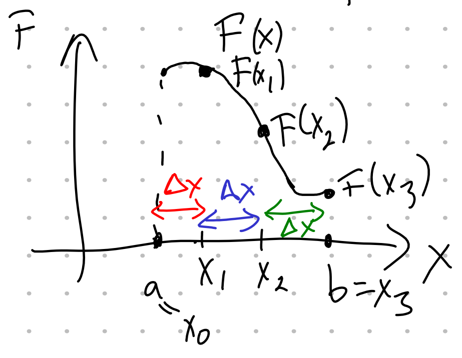
$$\begin{aligned} \text{Work} &= \text{Force} \times \text{displacement} \\ &= \text{Weight} \times 0.7 \\ &= mg \times 0.7 \\ &= 1.2 \times 9.8 \times 0.7 = 8.2\,(J) \end{aligned}$$

(b) Work = Force × displacement

= Weight × displacement

$= 20 \cdot 6 = 120\ ft\text{-}lb$

Q) What if the force changes throughout the motion? How do we find the work?



Total Work from a to b = The sum of work that is done as the object moves from a to x_1, x_1 to x_2, x_2 to b under the force $F(x)$.

$= W_1 + W_2 + W_3$

$W_1 \approx F(x_1)(x_1 - x_0)$

$W_2 \approx F(x_2)(x_2 - x_1)$

$W_3 \approx F(x_3)(x_3 - x_2)$

We can assume that $x_1 - x_0 = x_2 - x_1 = x_3 - x_2 = \Delta x$

$$W \approx \sum_{i=1}^{3} F(x_i)\Delta x$$

$$W = \lim_{n\to\infty} \sum_{i=1}^{n} F(x_i)\Delta x = \int_a^b F(x)\,dx$$

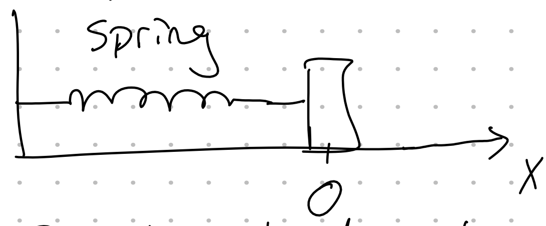
Example When a particle is located a distance x feet from the origin, a force of $x^2 + 2x$ pounds acts on it. How much work is done in moving it from $x=1$ to $x=3$?

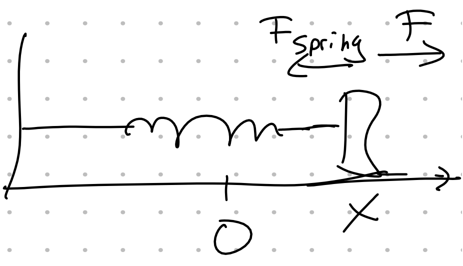
$$W = \int_1^3 (x^2 + 2x)\,dx = \frac{x^3}{3} + \frac{2x^2}{2}\Big|_1^3 = (9+9) - \left(\frac{1}{3} - 1\right)$$

$= \frac{50}{3}$ ft-lb

Hooke's Law

no friction



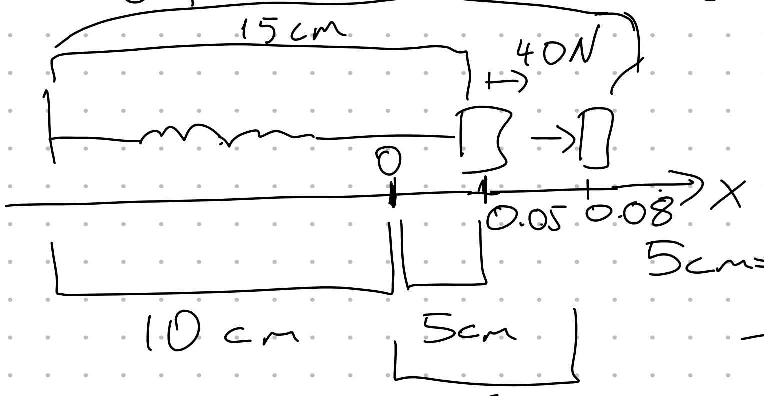


In its natural position the spring does not exert any force

$F = kx$

the spring constant

Example A force of 40N is required to hold a spring that has been stretched from its natural length of 10cm to a length of 15cm. How much work is done in stretching the spring from 15cm to 18cm?



5cm = 0.05m

$F = kx$

$40 = k\,0.05$

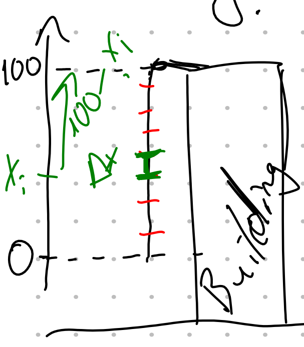
$k = \frac{40}{0.05} = \frac{4000}{5} = 800$

$$W = \int_{0.05}^{0.08} F(x)\,dx = \int_{0.05}^{0.08} kx\,dx = 800\int_{0.05}^{0.08} x\,dx$$

$$= \cancel{800}^{400}\frac{x^2}{\cancel{2}}\Big|_{0.05}^{0.08} = 400(0.08^2 - 0.05^2)$$

$$= 1.56\ J$$

Example A 200-lb cable is 100 ft long and hangs vertically from the top of a tall building. How much work is required to lift the cable to the top of the building?



We "cut" the cable into short pieces
Find the work required for each one of the pieces and add them up.

$W_i \approx$ Force $\cdot$ displacement
$= \text{Weight} \cdot (100 - x_i)$
$= \text{density} \cdot \text{length}_i \cdot (100 - x_i)$
$= (2\frac{lb}{ft}\,\Delta x\,\cancel{ft})\cdot(100 - x_i)(ft)$
$= 2\Delta x(100 - x_i)$ ft-lb

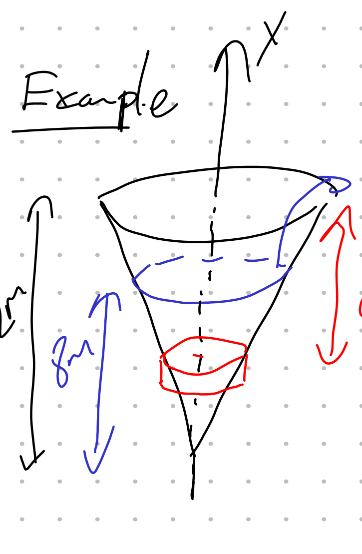
density $= \frac{200\ lb}{100\ ft} = 2\ lb/ft$

Total Work $\approx \sum_{i=1}^{n} W_i = \sum_{i=1}^{n} 2(100 - x_i)\Delta x$

Total Work $= \lim_{n\to\infty}\sum_{i=1}^{n} 2(100 - x_i)\Delta x = \int_0^{100} 2(100 - x)\,dx$

Exercise.

Example



pump all the water to the top of the tank.

displacement

We will continue next time.