WW #14

$$\int \frac{2\ln(\tan(x))}{\sin(x)\cos(x)}dx$$

$u = \tan x \qquad du = \sec^2 x\, dx$

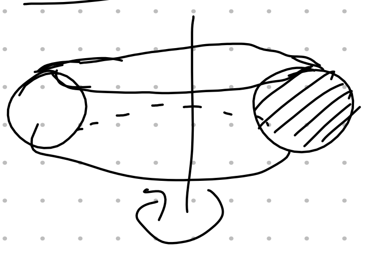
$\sec x = \dfrac{1}{\cos x}$

$$= \int \frac{2\ln(\tan x)}{\frac{\sin x}{\cos x}\cdot \cos^2 x}dx = \int \frac{2\ln(\tan x)}{\tan x}\sec^2 x\, dx$$

$$= \int \frac{2\ln(u)}{u}du \qquad v = \ln(u) \qquad dv = \frac{1}{u}du$$

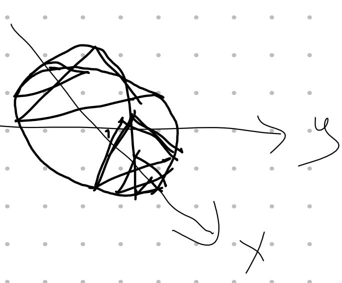
$$= \int 2v\, dv = \cancel{2}\frac{v^2}{\cancel{2}} + C = (\ln u)^2 + C$$

Last time

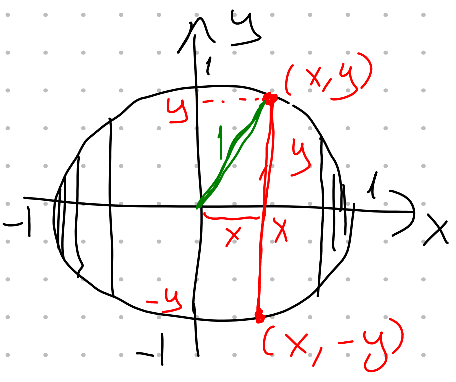


Solids of revolution

Example The following figure shows a solid with a circular base of radius 1. Parallel cross sections perpendicular to the x-axis are equilateral triangles. Find the volume of the solid.



$$V = \int_a^b A(x)\,dx$$



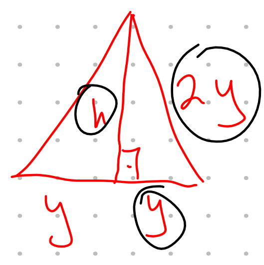
$a = -1 \qquad b = 1$

$A(x) =$ Area of the equilateral triangle of side $y-(-y) = 2y$

$\boxed{1 = x^2 + y^2}$

$y^2 = 1 - x^2$

$y = \sqrt{1-x^2}$



$(2y)^2 = h^2 + y^2 \rightsquigarrow 4y^2 = h^2 + y^2$

$h^2 = 3y^2 \rightsquigarrow h = \sqrt{3}\,y$

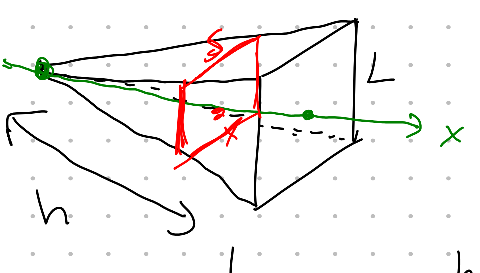
$A = \frac{1}{2}bh = \frac{1}{\cancel{2}}(\cancel{2}y)\sqrt{3}y = \sqrt{3}y^2$

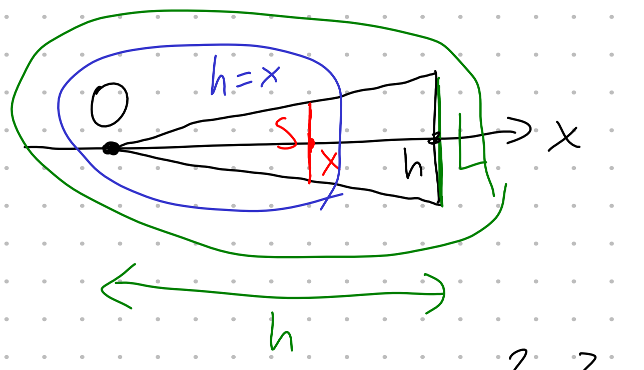
$A(x) = \sqrt{3}\,y^2 = \sqrt{3}(1-x^2)$

$$V = \int_{-1}^{1}\sqrt{3}(1-x^2)\,dx = \sqrt{3}\int_{-1}^{1}(1-x^2)\,dx = 2\sqrt{3}\int_0^1 (1-x^2)\,dx$$

$$= 2\sqrt{3}\left(x - \frac{x^3}{3}\right)\Big|_0^1 = 2\sqrt{3}\left(1-\frac{1}{3}\right) = \frac{4}{3}\sqrt{3}$$

Example Find the volume of a pyramid whose base is a square with side L and whose height is h.





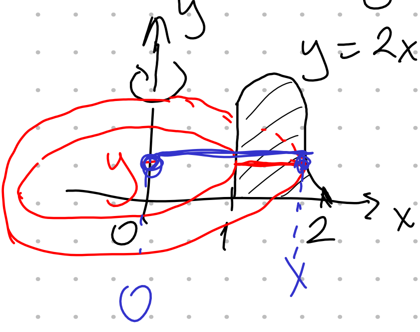
$$V = \int_a^b A(x)\,dx = \int_0^h A(x)\,dx \qquad A(x) = s^2 = \frac{L^2x^2}{h^2}$$

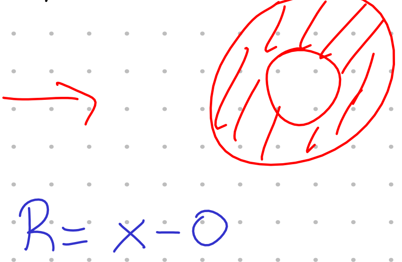
similar triangles: $\boxed{\dfrac{s}{x} = \dfrac{L}{h}} \rightsquigarrow s = \dfrac{Lx}{h}$

$$V = \int_0^h \frac{L^2x^2}{h^2}dx = \frac{L^2}{h^2}\int_0^h x^2\,dx = \frac{L^2}{h^2}\frac{x^3}{3}\Big|_0^h = \frac{L^2}{h^2}\frac{h^3}{3}$$

$$= \frac{L^2h}{3}$$

6.3 Volumes by Cylindrical Shells



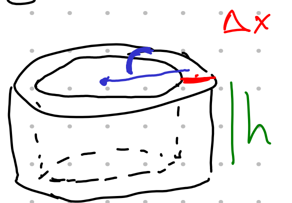


this is a washer but it is difficult to find its outer radius.

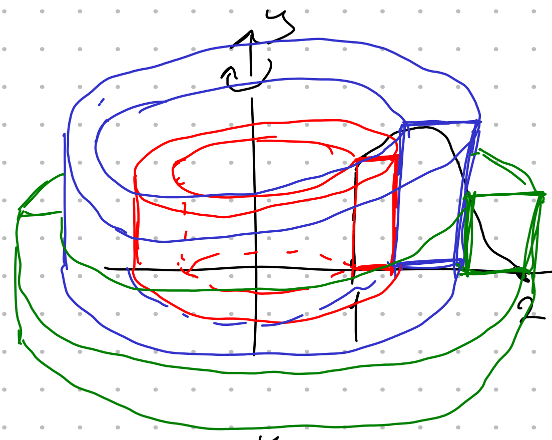
$R = x - 0$

solving $y = 2x^2 - x^3$ for x is very difficult. In general, it may not be possible.

Cylindrical shells:

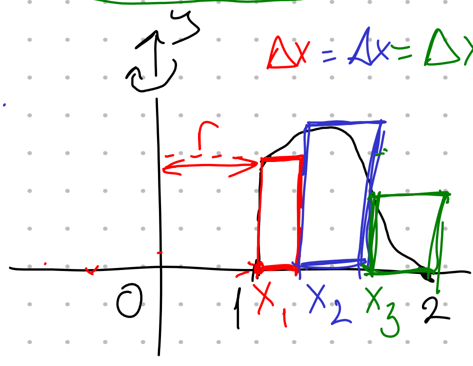


$V = 2\pi r h \Delta x$



Volume $\approx$ Volume of Cyl. shell 1 + Volume of Cyl. shell 2 + Volume of Cyl. shell 3

$= 2\pi r_1 h_1 \Delta x = 2\pi x_1 f(x_1)\Delta x$, $\quad 2\pi x_2 f(x_2)\Delta x$, $\quad 2\pi x_3 f(x_3)\Delta x$

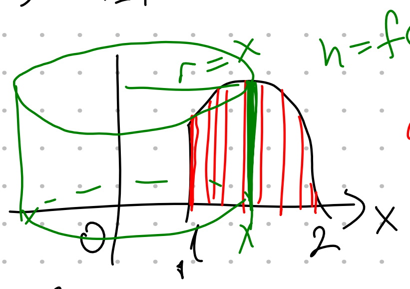


$h_1 = f(x_1) \quad r_1 = x_1$

$h_2 = f(x_2) \quad r_2 = x_2$

$h_3 = f(x_3) \quad r_3 = x_3$

$$\text{Vol} = \lim_{n\to\infty} \sum_{i=1}^{n} 2\pi x_i f(x_i)\Delta x = \int_a^b 2\pi x f(x)\,dx$$



$a = 1 \quad b = 2$

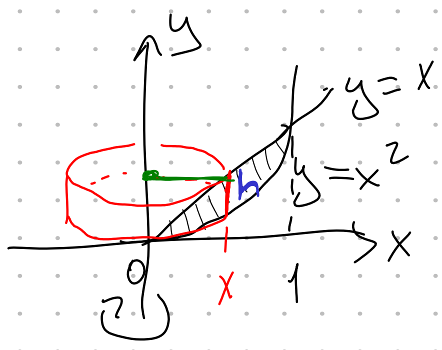
$V = \int_a^b A(x)\,dx$

$A(x) = (2\pi r)\,h$

$$V = \int_1^2 2\pi x(2x^2 - x^3)\,dx = 2\pi \int_1^2 (2x^3 - x^4)\,dx$$

$$= 2\pi\left(\frac{2x^4}{4} - \frac{x^5}{5}\right)\Bigg|_1^2 = 2\pi\left(8 - \frac{32}{5}\right) - 2\pi\left(\frac{1}{2} - \frac{1}{5}\right)$$

Example Find the volume of the solid obtained by rotating about the y-axis the region between $y = x$ and $y = x^2$.



$V = \int_a^b A(x)\,dx$

$a = 0 \quad b = 1$

$A(x) = 2\pi r h$

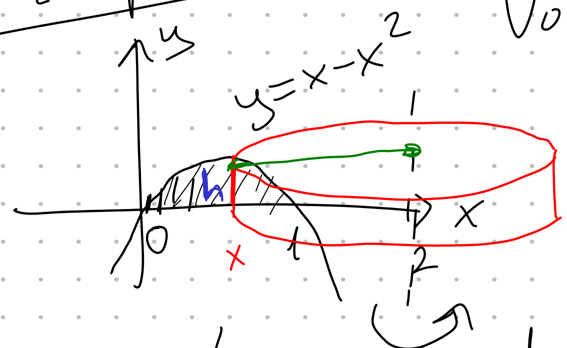
$r = x - 0 = x$

$h = x - x^2$

$$V = \int_0^1 2\pi r h\,dx = \int_0^1 2\pi x(x - x^2)\,dx = 2\pi \int_0^1 (x^2 - x^3)\,dx$$

Exercise.

Example



Volume?

$A(x) = 2\pi r h$

$r = 2 - x$

$h = x - x^2$

$$V = \int_a^b A(x)\,dx = \int_0^1 2\pi(2-x)(x - x^2)\,dx$$

Exercise