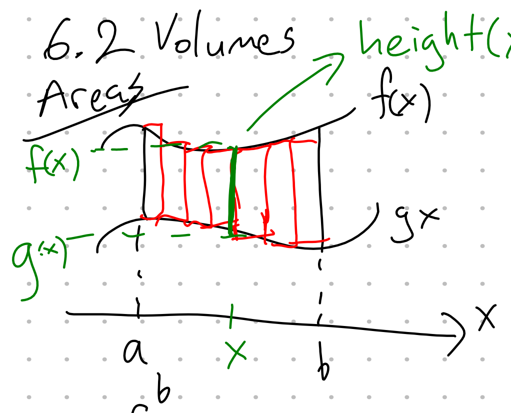
6.2 Volumes

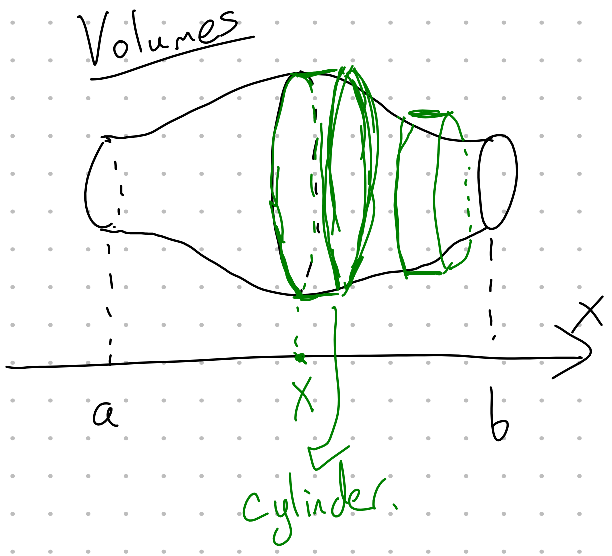
Areas



$$A=\int_a^b (f(x)-g(x))\,dx$$

$$=\int_a^b \text{height}(x)\,dx$$

Volumes



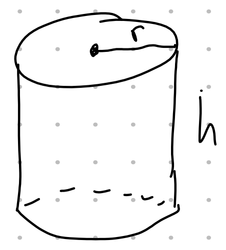
What are cylinders?

B_2 is a copy of B_1 in a parallel plane

the base of the cylinder

The set of all points between the base B_1 and a copy of it in a parallel plane B_2 is called a cylinder.

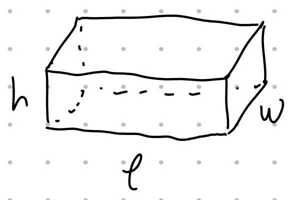
Volume = height × Area of base = hB_1



"Circular" cylinder its base is a circle

$V = h\pi r^2$

$= \text{height} \times (\text{Area of base})$

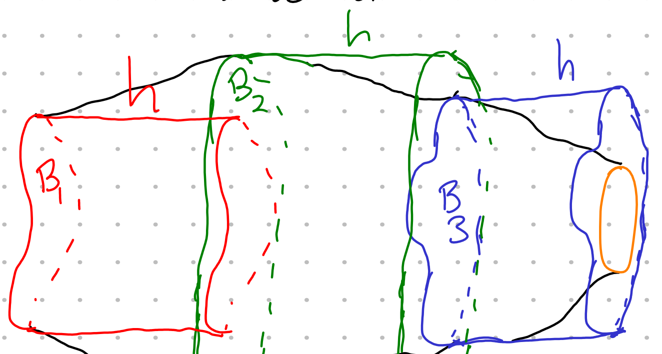


Rectangular box is also a cylinder

$V = h\ell w = h(\ell w)$

$= \text{height} \times (\text{Area of base})$

Slice = Cross section



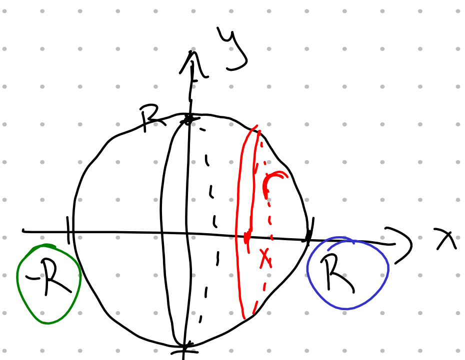
Volume ≈ Volume of Cylinder 1 + Volume of Cylinder 1 + Volume of Cylinder 1

$= hB_1 + hB_2 + hB_3$

Definition of Volume Let S be a solid that lies between $x=a$ and $x=b$. If the cross-sectional area of S in the plane P_x, through x and perpendicular to the x-axis, is $A(x)$ then the volume is

$$V=\lim_{n\to\infty}\sum_{i=1}^{n} A(x_i^*)\Delta x=\int_a^b A(x)\,dx$$

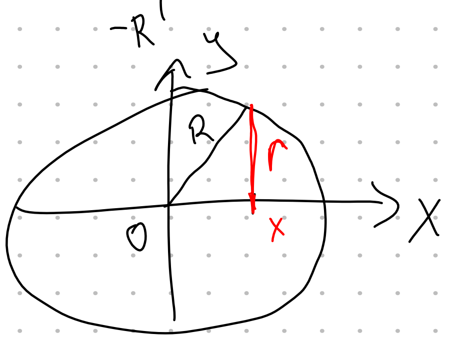
Example Show that the volume of a sphere of radius R is $V=\frac{4}{3}\pi R^3$



$$V=\int_a^b A(x)\,dx$$

$a=-R$ $\quad$ $b=R$

$A(x)=\pi r^2$



$R^2 = r^2 + x^2$

$\hookrightarrow r^2 = R^2 - x^2$

$r = \sqrt{R^2-x^2}$

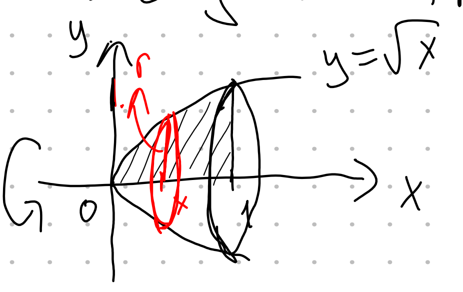
$A(x) = \pi r^2 = \pi(R^2-x^2)$

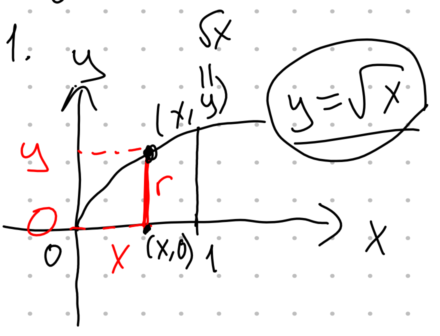
$$V=\int_a^b A(x)\,dx=\int_{-R}^{R}\pi(R^2-x^2)\,dx=\pi\int_{-R}^{R}(R^2-x^2)\,dx$$

$$=\pi\left(R^2x-\frac{x^3}{3}\right)\Big|_{-R}^{R}=\pi\left(R^3-\frac{R^3}{3}-\left(-R^3+\frac{R^3}{3}\right)\right)$$

$$=\pi\left(\frac{2}{3}R^3+\frac{2}{3}R^3\right)=\frac{4}{3}\pi R^3$$

Example Find the volume of the solid obtained by rotating about the x-axis the region under the curve $y=\sqrt{x}$ from 0 to 1.

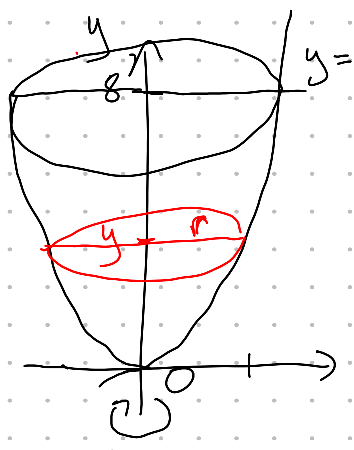




$$V=\int_a^b A(x)\,dx=\int_0^1 \pi r^2\,dx \qquad r=y-0=\sqrt{x}-0=\sqrt{x}$$

$$=\int_0^1 \pi(\sqrt{x})^2\,dx=\pi\int_0^1 x\,dx=\pi\frac{x^2}{2}\Big|_0^1=\frac{\pi}{2}$$

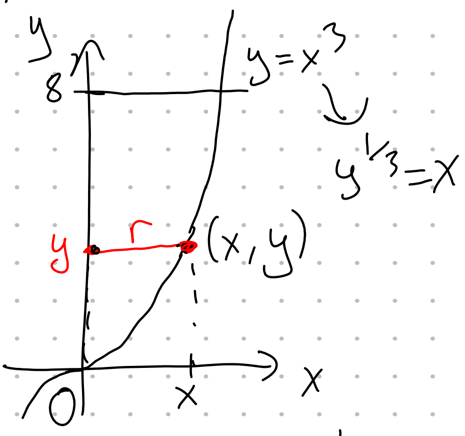
Example. Find the volume of the solid obtained by rotating the region bounded by $y=x^3$, $y=8$, and $x=0$ about the y-axis.



$$V=\int_c^d A(y)\,dy$$

$$c=0 \quad d=8$$

$$A(y)=\pi r^2$$

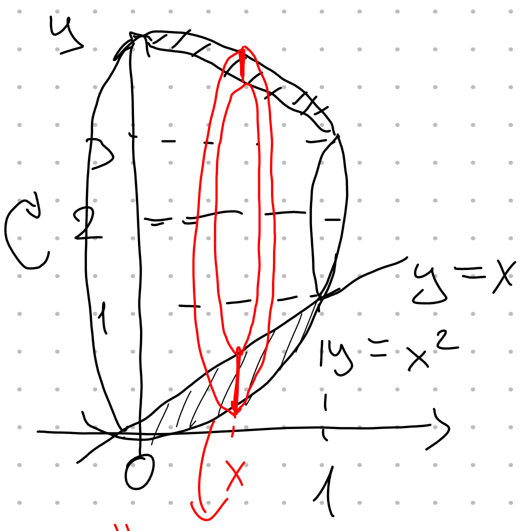


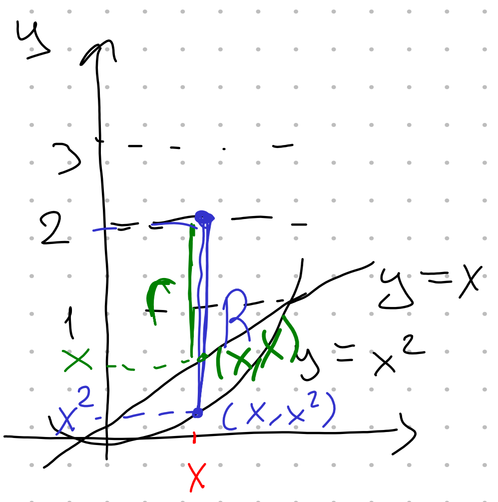
$$r=x-0=y^{1/3}-0=y^{1/3}$$

$$V=\int_0^8 \pi r^2\,dy=\pi\int_0^8 (y^{1/3})^2\,dy=\pi\int_0^8 y^{2/3}\,dy$$

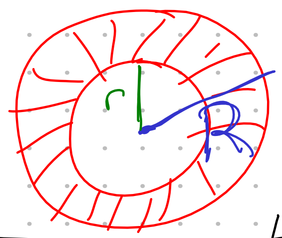
$$=3\pi\frac{y^{5/3}}{5}\Big|_0^8=\frac{96\pi}{5}$$

(*) Example. The region R enclosed by $y=x$ and $y=x^2$ is rotated about the line $y=2$. Find the volume.





"Washes"



$$\boxed{A=\pi R^2-\pi r^2}$$

$$R=2-x^2$$

$$r=2-x$$

$$V=\int_a^b A(x)\,dx$$

$$=\int_0^1\left(\pi(2-x^2)^2-\pi(2-x)^2\right)dx$$