Math 162 Calculus 2

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Office hours: Mon, Wed 3:30-5:00 pm (Starting this Wednesday)

Zoom link on Blackboard (same link as the lecture)

You can attend office hours of any other MTH 162 professor or TA.

Textbook: Calculus: Early Transcendentals (9th edition) by James Stewart

Grade:

- Webwork HW 15% Every week due on Friday 11:59 pm
- Workshop attendance and participation 10%
- 3 Midterm exams 25% each
- Final exam 25%

(Webcam Required during exams and workshops)

Lowest MT score will be dropped.
Lowest WW-HW will be dropped.
Lowest 2 Workshop scores will be dropped

No WW-Quizzes this semester!

To access WW, you need to click on the link on Blackboard.
WW-Set 0 is a warmup set due this Friday, Feb 5.

Workshops will start next week. Signup will be available this Wed, Feb 3 at 6:30 pm.

Recall FTC 1

If $g(x) = \int_a^x f(t)\,dt$ then $g'(x) = f(x)$

FTC 2

If $F' = f$ then $\int_a^b f(x)\,dx = F(b) - F(a)$

u-substitution

$$\int f(g(x))\, g'(x)\,dx = \int f(u)\,du \qquad u = g(x), \quad du = g'(x)\,dx$$

Example If $F(x) = \int_3^x \frac{1}{1+t^3}\,dt$, $F'(x) = ?$

$$F'(x) = \frac{1}{1+x^3}$$

Example $h(x) = \int_{-5}^{\sin(x)} (\cos(t^4) + t)\,dt$ $h'(x) = ?$

Set $g(x) = \int_{-5}^{x} (\cos(t^4) + t)\,dt$ then $\boxed{h(x) = g(\sin x)}$

By FTC1, $g'(x) = \cos(x^4) + x$

By the Chain Rule, $h'(x) = g'(\sin x)\,(\sin x)'$

Thus, $h'(x) = (\cos(\sin^4 x) + \sin x)\cos x$

Example Given $f(x) = \begin{cases} -9x & \text{if } x \le 0 \\ \sin x & \text{if } x > 0 \end{cases}$

find $\int_{-\pi}^{\pi} f(x)\,dx = \int_{-\pi}^{0} f(x)\,dx + \int_0^{\pi} f(x)\,dx$

$$= -9\int_{-\pi}^{0} x\,dx + \int_0^{\pi} \sin x\,dx = -9\,\frac{x^2}{2}\Big|_{-\pi}^{0} - \cos x\Big|_0^{\pi}$$

$$= 0 - \left(-\frac{9}{2}\pi^2\right) - \cos\pi - (-\cos 0)$$

$$= \frac{9}{2}\pi^2 + 1 + 1 = \frac{9}{2}\pi^2 + 2$$

Example $\int_0^{\pi/2} \cos x \sin(\sin x)\,dx$ $\qquad u = \sin x, \quad du = \cos x\,dx$

$$= \int_0^1 \sin u\,du$$

when $x = \frac{\pi}{2}$, $u = \sin\frac{\pi}{2} = 1$

when $x = 0$, $u = \sin 0 = 0$

$$= -\cos u\Big|_0^1 = -\cos 1 - (-\cos 0) = -\cos 1 + 1$$

$$\int_a^b \sin u\,du = -\cos u\Big|_a^b = -\cos(\sin(x))\Big|_0^{\pi/2}$$

e.g. $h(x) = \int_x^{\sin x} t^5\,dt = \int_x^0 t^5\,dt + \int_0^{\sin x} t^5\,dt$

$$= -\int_0^x t^5\,dt + \int_0^{\sin x} t^5\,dt$$

Example $\int \frac{1-\sin^2 x}{\cos x}\,dx$ $\qquad \cos^2 x + \sin^2 x = 1$

$$= \int \frac{\cos^2 x}{\cos x}\,dx = \int \cos x\,dx = \sin x + C$$

Example $\int \frac{3x\,dx}{1+x^4}$ let's try $u = 1+x^4$

$du = 4x^3\,dx$

$du = 4x^2 \, x\,dx$

$u-1 = x^4$

$\sqrt{u-1} = x^2$

We can do it but there is an easier way!

$$\int \frac{3x\,dx}{1+x^4}$$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$= \frac{3}{2}\int \frac{du}{1+u^2}$$

$$= \frac{3}{2}\arctan(u) + C = \frac{3}{2}\arctan(x^2) + C$$

Example $\int \frac{x^3}{\sqrt{x-1}}\,dx$

$u = x-1$

$du = dx$

$x = u+1$

$$= \int \frac{(u+1)^3}{\sqrt{u}}\,du$$

$$= \int (u+1)^3 u^{-1/2}\,du = \int (u^3+3u^2+3u+1)\,u^{-1/2}\,du$$

$$= \int \left(u^{5/2} + 3u^{3/2} + 3u^{1/2} + u^{-1/2}\right) du$$

$$= \frac{2}{7}u^{7/2} + 3\left(\frac{2}{5}u^{5/2}\right) + 3\left(\frac{2u^{3/2}}{3}\right) + 2u^{1/2} + C$$

$$= \frac{2}{7}(x-1)^{7/2} + \frac{6}{5}(x-1)^{5/2} + 2(x-1)^{3/2} + 2(x-1)^{1/2} + C$$

e.g. $\int \frac{x^3}{\sqrt{x^2-1}}\,dx$

$u = x^2-1$

$du = 2x\,dx$

$$= \int \frac{x^2 \; x\,dx}{\sqrt{x^2-1}} = \frac{1}{2}\int \frac{(u+1)}{\sqrt{u}}\,du$$