WW13 counts towards your grade but WW14 is optional.

P18 Find the radius of conv.

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt{n+3}} \qquad a_n = \frac{(-1)^n x^n}{\sqrt{n+3}}$$

$$\left|\frac{a_{n+1}}{a_n}\right| = \left| \frac{\frac{x^{n+1}}{\sqrt{n+4}}}{\frac{x^n}{\sqrt{n+3}}} \right| = \left| \frac{x^{n+1}}{\cancel{x^n}} \cdot \frac{\sqrt{n+3}}{\sqrt{n+4}} \right| = |x| \cdot \sqrt{\frac{n+3}{n+4}}$$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} |x| \sqrt{\frac{n+3}{n+4}} = |x| \sqrt{\lim_{n\to\infty} \frac{n+3}{n+4}} = |x|$$

So if $|x|<1$, the series is conv and if $|x|>1$ it is DIV.

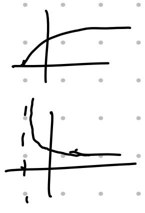
$R = 1$. What about $|x| = 1$?

If $x = 1$, $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+3}}$ $\quad a_n = (-1)^n \cdot \frac{1}{\sqrt{n+3}}$ $\quad |a_n| = \frac{1}{\sqrt{n+3}}$ dec ✓

alternating series ✓ $\quad |a_n| \to 0$ ✓

$f(x) = x+3$ inc.

$g(x) = \sqrt{f(x)}$ inc.



$h(x) = \frac{1}{g(x)} = \frac{1}{\sqrt{f(x)}} = \frac{1}{\sqrt{x+3}}$ dec.

So by the alternating series test, this series is CONV. at $x=1$.

If $x=-1$, $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+3}} \sim \frac{1}{\sqrt{n}} = \frac{1}{n^{1/2}}$ $\quad \sum \frac{1}{n^p}$ $\quad p = \frac{1}{2} \le 1$ DIV

$$\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n+3}}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty} \sqrt{\frac{n}{n+3}} = 1 \neq 0$$

So, by the limit comparison test $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+3}}$ is DIV since $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ is DIV.

So the Int. of Conv. is $(-1, 1]$

P20 $\quad \sum_{n=1}^{\infty} \frac{(7x-3)^n}{n^2}$. $\quad a_n = \frac{(7x-3)^n}{n^2}$ $\quad \left|\frac{a_{n+1}}{a_n}\right| = \left| \frac{(7x-3)^{n+1}}{(7x-3)^n} \cdot \frac{n^2}{(n+1)^2} \right|$

$\left|\frac{a_{n+1}}{a_n}\right| = |7x-3| \left(\frac{n}{n+1}\right)^2$ $\quad$ so $\lim_{n\to\infty} |7x-3| \left(\frac{n}{n+1}\right)^2 = |7x-3|$

$|7x-3| < 1 \rightarrow \frac{|7x-3|}{7} < \frac{1}{7} \rightarrow |x - \frac{3}{7}| < \frac{1}{7} = R$

CONV. if by Ratio test

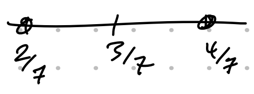
Boundary case when $|7x-3| = 1$ $\quad |x - \frac{3}{7}| = \frac{1}{7}$ or

Say $7x-3 = 1$

$$\sum_{n=1}^{\infty} \frac{1^n}{n^2} = \sum_{n=1}^{\infty} \frac{1}{n^2}$$ $p = 2 > 1$ so it is CONV at

If $7x-3 = -1$ $\quad \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ is ABS. CONV. since $\sum \left|\frac{(-1)^n}{n^2}\right|$ is the previous series.

So this series is also CONV.



Int. of Conv is $[\frac{2}{7}, \frac{4}{7}]$.

After Class:

$1 - \frac{1}{10} + \frac{1}{100} - \frac{1}{1000} + \cdots$ How many terms do you have to use for your approximation to be within $10^{-7}$ from the convergent value.

Alternating series $\Rightarrow$ $|\text{Error}| \le |\text{the first term omitted}|$

$$\underbrace{10^0 - 10^{-1} + 10^{-2} - 10^{-3} + \cdots + 10^{-6}}_{7 \text{ terms}} / - 10^{-7}$$