<u>Example</u> $\sum_{n=1}^{\infty}\left(\frac{3}{n(n+1)}+\frac{1}{2^n}\right)=\sum_{n=1}^{\infty}\frac{3}{n(n+1)}+\sum_{n=1}^{\infty}\frac{1}{2^n}$

$$=\underbrace{3\sum_{n=1}^{\infty}\frac{1}{n(n+1)}}+\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots\right)$$

$a+ar+ar^2+\cdots$

$a=\frac{1}{2}$ $\quad ar=\frac{1}{4}$ $\quad r=\frac{1}{2}$

$\frac{1}{n(n+1)}=\frac{1}{n}-\frac{1}{n+1}$

$$3\left[\left(\frac{1}{1}-\cancel{\frac{1}{2}}\right)+\left(\cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}\right)+\cancel{\frac{1}{3}}-\cdots\right] \qquad \frac{a}{1-r}=\frac{\frac{1}{2}}{1-\frac{1}{2}}=1$$

$$=3+1=4$$

<u>Example</u> $\sum_{n=1}^{\infty}1$ DIV $\quad \sum_{n=1}^{\infty}(n+1)-n=(\cancel{2}-1)+(\cancel{3}-\cancel{2})+(\cancel{4}-\cancel{3})+\cdots$

$$=-1$$

$$s_n=\sum_{i=1}^{n}(i+1)-i=(\cancel{2}-1)+(\cancel{3}-\cancel{2})+(\cancel{4}-\cancel{3})+\cdots+[(n+1)-\cancel{n}]$$

$$=n+1-1=n$$

$s_n=n\to\infty$ as $n\to\infty$

<u>Example</u> $\sum_{n=1}^{\infty}\frac{\ln n}{n}$ CONV or DIV?

$f(x)=\frac{\ln x}{x}$ $\qquad \int_1^{\infty}\frac{\ln x}{x}\,dx$ $\qquad u=\ln x \quad du=\frac{1}{x}dx$

$$=\int_0^{\infty}u\,du=\frac{u^2}{2}\Big|_0^{\infty}=\lim_{t\to\infty}\left(\frac{t^2}{2}-0\right)=\infty$$

DIV

Hence $\sum_{n=1}^{\infty}\frac{\ln n}{n}$ is also DIV.

Alt. $\frac{\ln x}{x}\geq\frac{1}{x}$ for $x\geq e$ $\qquad \underset{\text{DIV}}{\int_e^{\infty}\frac{\ln x}{x}dx}\geq\underset{\text{DIV}}{\int_e^{\infty}\frac{1}{x}dx}$