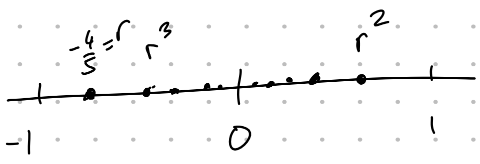
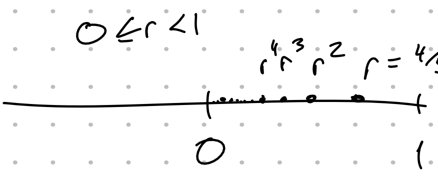
$-1<r<0 \quad 0\le r^n$



$r^2=\frac{4^2}{5^2}$

$0\le r<1$



$|r|<1$

$\Rightarrow \quad r^n\to 0$ as $n\to\infty$

Example Show that $a_n=\frac{n}{n^2+1}$ is decreasing

Method I: $a_n > a_{n+1} \quad \to \quad \frac{n}{n^2+1} > a_{n+1}=\frac{n+1}{(n+1)^2+1}$

$$\frac{n}{n^2+1} > \frac{n+1}{n^2+2n+2} \qquad n(n^2+2n+2) > (n^2+1)(n+1)$$

$$\cancel{n^3}+\cancel{2}n^2+\cancel{2}n > \cancel{n^3}+\cancel{n^2}+\cancel{n}+1$$

$$n^2+n>1 \quad \text{since } n\ge 1 \ \checkmark$$

So $a_n > a_{n+1}$ must also be true.

Method II: $a_n=\frac{n}{n^2+1}$ Consider $f(x)=\frac{x}{x^2+1}$ so $a_n=f(n)$

We can show that f is decreasing. So we need to take derivative and show that it is less than 0.

$f'(x)<0$.

Example Given $a_1=2$ $a_{n+1}=\frac{1}{2}(a_n+6)$ for $n\ge 1$ determine $\lim_{n\to\infty} a_n$ if it exists.

$a_1=2$

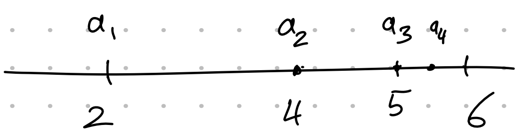
$a_2=\frac{1}{2}(a_1+6)=\frac{1}{2}(2+6)=4$

$a_3=\frac{1}{2}(a_2+6)=\frac{1}{2}(4+6)=5$

a_n seems to be increasing.

a_{n+1} is the average of a_n and 6.

$a_{n+1}=\frac{a_n+6}{2}$



So a_n is indeed increasing and $a_n<6$ for $n\ge 1$

a_n is monotonic (b/c it is increasing) and it is bounded above. Since it is increasing, it is bdd below as well. So by the Monotonic Sequence Thm. it has a limit. Say $\lim_{n\to\infty} a_n = L$.

$$L=\lim_{n\to\infty} a_n=\lim_{n\to\infty} a_{n+1}=\lim_{n\to\infty}\frac{1}{2}(a_n+6)$$

$$=\frac{1}{2}\left(\lim_{n\to\infty} a_n + 6\right)$$

$$L=\frac{1}{2}(L+6)$$

$$\cancel{2}L=\cancel{L}+6$$

$$L=6$$

$a_1=2$ $\quad a_{n+1}=\frac{1}{2}(a_n+6)$ $\quad n\ge 1$

a_{n+1} is the average of a_n and 6