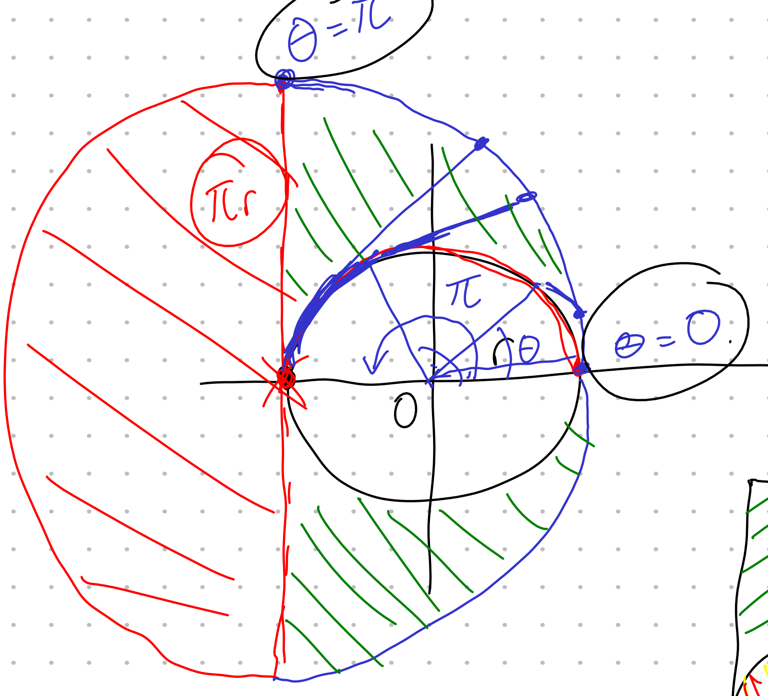
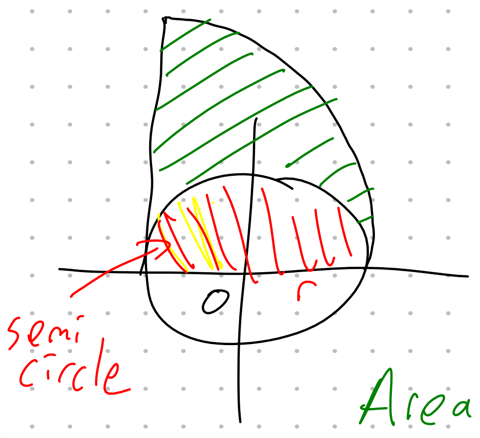
WW 8 #13

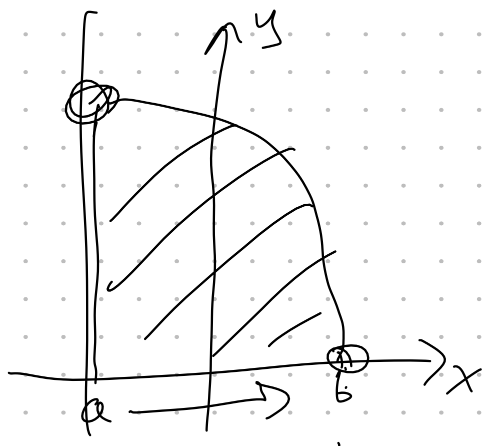


$x = r(\cos\theta + \theta\sin\theta)$

$y = r(\sin\theta - \theta\cos\theta)$

Area $= \dfrac{\pi(\pi r)^2}{2} = \dfrac{\pi^3 r^2}{2}$





Area = area under the curve − area of semi circle ($\frac{1}{2}\pi r^2$)

$y = F(x)$

$$A = \int_a^b F(x)\,dx = \int_a^b y\,dx$$

$$= \int_\pi^0 r(\sin\theta - \theta\cos\theta)\, r\theta\cos\theta\, d\theta$$

$x = r(\cos\theta + \theta\sin\theta)$

$y = r(\sin\theta - \theta\cos\theta)$

$dx = r(-\sin\theta + \sin\theta + \theta\cos\theta)d\theta$

$= r\theta\cos\theta\, d\theta$

$$= -r^2\int_0^\pi (\theta\sin\theta\cos\theta - \theta^2\cos^2\theta)\,d\theta$$

$\sin\theta\cos\theta = \dfrac{\sin 2\theta}{2}$

$$\int \theta\sin\theta\cos\theta\, d\theta = \frac{1}{2}\int \theta\sin 2\theta\, d\theta$$

$u = \theta$ $\quad dv = \sin 2\theta\, d\theta$

$du = d\theta$ $\quad v = -\dfrac{\cos 2\theta}{2}$

$uv - \int v\,du$

$$= \frac{1}{2}\left[\frac{-\theta\cos 2\theta}{2} + \frac{1}{2}\int \cos 2\theta\, d\theta\right] = -\frac{\theta\cos 2\theta}{4} + \frac{1}{8}\sin 2\theta + C$$

$$-r^2\int_0^\pi \theta\sin\theta\cos\theta\, d\theta = -r^2\left(-\frac{\theta\cos 2\theta}{4} + \frac{1}{8}\sin 2\theta\right)\Big|_0^\pi$$

$$= -r^2\left(-\frac{\pi(1)}{4} + 0\right) - \left(-r^2\left(\frac{0}{4} + 0\right)\right)$$

$$= \frac{\pi r^2}{4}$$

$\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

$$-r^2\int_0^\pi -\theta^2\cos^2\theta\, d\theta = +\frac{r^2}{2}\int_0^\pi \theta^2(1 + \cos 2\theta)\,d\theta$$

$$= \frac{r^2}{2}\int_0^\pi \theta^2\, d\theta + \frac{r^2}{2}\int_0^\pi \theta^2\cos 2\theta\, d\theta$$

$u = \theta^2$ $\quad dv = \cos 2\theta\, d\theta$

$du = 2\theta\, d\theta$ $\quad v = \dfrac{\sin 2\theta}{2}$

$$\frac{r^2}{2}\frac{\theta^3}{3}\Big|_0^\pi = \frac{\pi^3 r^2}{6}$$

$$= \frac{r^2}{2}\left(\frac{\theta^2\sin 2\theta}{2}\Big|_0^\pi - \int_0^\pi \theta\sin 2\theta\, d\theta\right)$$

0

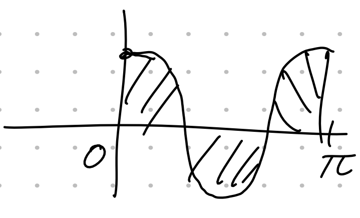
$u = \theta$ $\quad dv = \sin 2\theta\, d\theta$

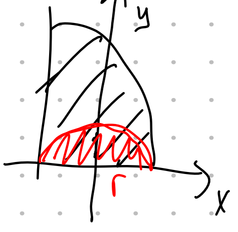
$du = d\theta$ $\quad v = -\dfrac{\cos 2\theta}{2}$

$$-\frac{r^2}{2}\int_0^\pi \theta\sin 2\theta\, d\theta = -\frac{r^2}{2}\left[-\frac{\theta\cos 2\theta}{2}\Big|_0^\pi + \frac{1}{2}\int_0^\pi \cos 2\theta\, d\theta\right]$$

0

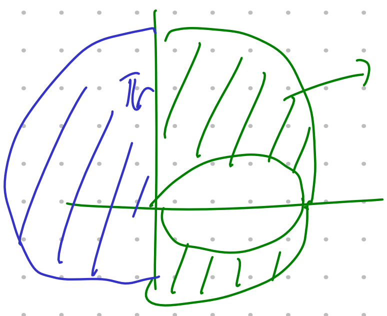
$$= \frac{r^2}{4}\pi(1) = \frac{\pi r^2}{4}$$





$$\frac{\pi r^2}{4} + \frac{\pi^3 r^2}{6} + \frac{\pi r^2}{4} = \frac{\pi^3 r^2}{6} + \frac{\pi r^2}{2}$$

$-\frac{1}{2}\pi r^2$



$A = \dfrac{\pi^3 r^2}{6}$

$2A = \dfrac{\pi^3 r^2}{3}$

$$\frac{1}{2}\pi(\pi r)^2 + \frac{\pi^3 r^2}{3} = \frac{\pi^3 r^2}{2} + \frac{\pi^3 r^2}{3} = \frac{5\pi^3 r^2}{6}$$