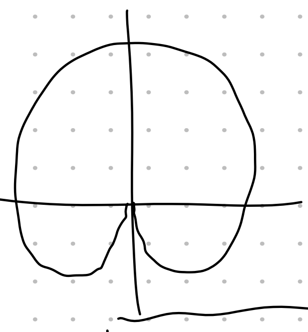
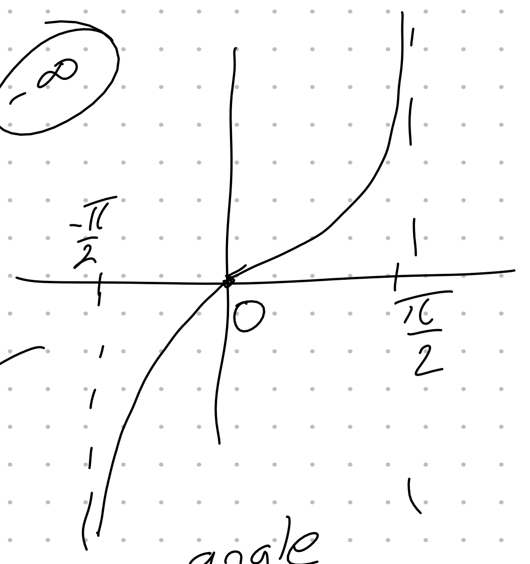
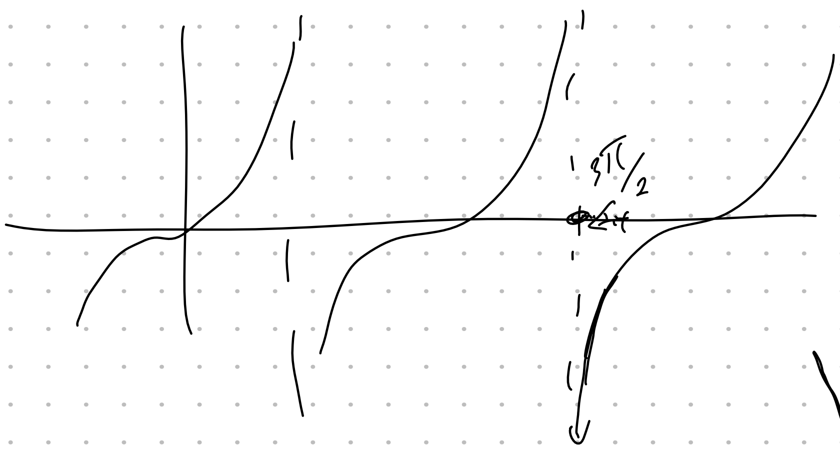
$r = 1+\sin t$



$$\lim_{\theta \to \frac{3\pi}{2}^+} \frac{dy}{dx} = \frac{1}{3} \boxed{\lim_{\theta \to \frac{3\pi}{2}^+} \tan\theta} = -\infty$$

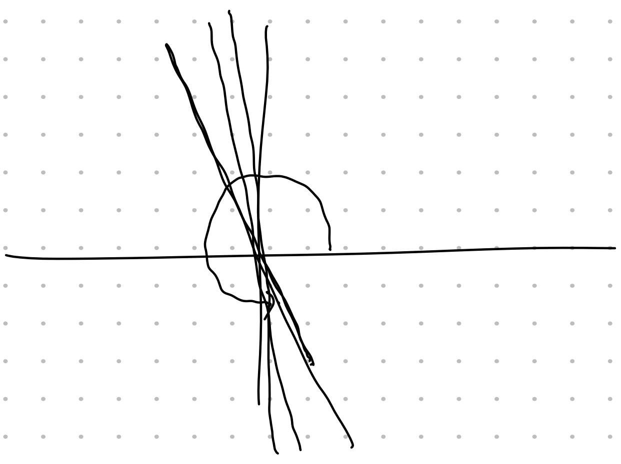




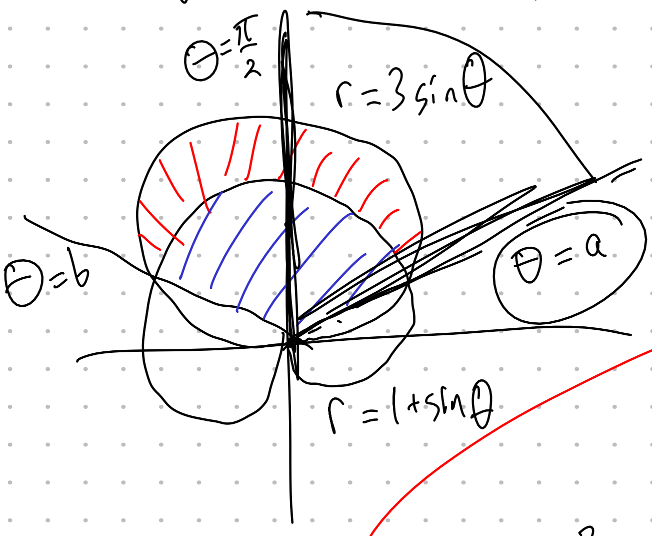
$\tan\theta$ ← angle

slope

$\theta \to \frac{3\pi}{2}^+$



Example Find the area of the region that lies inside circle $r=3\sin\theta$ and outside the cardioid $r=1+\sin\theta$



Find the area for $r=3\sin\theta$ from $\theta=a$ to $\theta=b$ − find the area for $r=1+\sin\theta$ $\theta=a$ to $\theta=b$

To find a we intersect the 2 curves

$r = 3\sin\theta = 1+\sin\theta$ so $2\sin\theta = 1$ $\quad \sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\theta = \frac{\pi}{6}$ $= a$

$$\frac{1}{2}\int_a^b r^2\,d\theta = 2\left(\frac{1}{2}\right)\int_a^{\pi/2} r^2\,d\theta$$

$$\frac{1}{2}\int_a^b r^2\,d\theta = 2\left(\frac{1}{2}\right)\int_a^{\pi/2} r^2\,d\theta$$

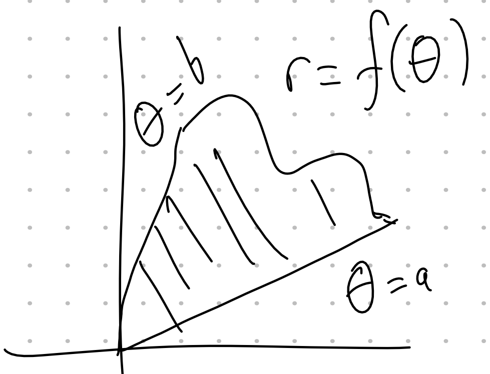
$$\int_{\pi/6}^{\pi/2} (3\sin\theta)^2\,d\theta - \int_{\pi/6}^{\pi/2} (1+\sin\theta)^2\,d\theta = \int_{\pi/6}^{\pi/2} \left(\underbrace{9\sin^2\theta - 1 - 2\sin\theta - \sin^2\theta}_{8\sin^2\theta}\right) d\theta$$

$$\boxed{\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)}$$

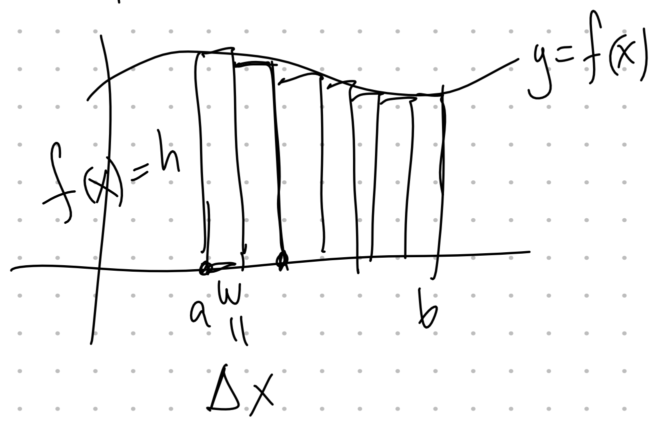
$$= \int_{\pi/6}^{\pi/2} \left(4(1-\cos 2\theta) - 1 - 2\sin\theta\right) d\theta$$

$$= \int_{\pi/6}^{\pi/2} (3 - 4\cos 2\theta - 2\sin\theta)\,d\theta = 3\theta - 2\sin 2\theta + 2\cos\theta \Big|_{\pi/6}^{\pi/2}$$

$= \pi$

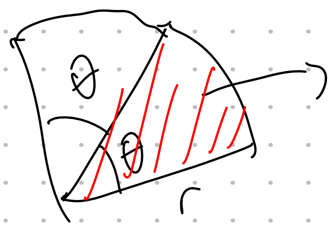


$$\text{Area} = \frac{1}{2}\int_a^b r^2\,d\theta$$



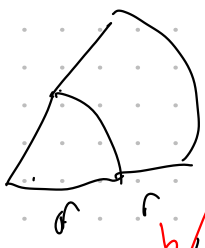
Area of Single Rect $= f(x)\Delta x$ $\quad$ Total Area $\simeq \sum f(x)\Delta x$

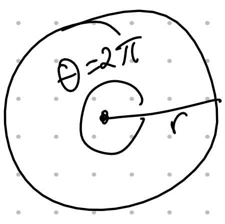
$\int f(x)\,dx$



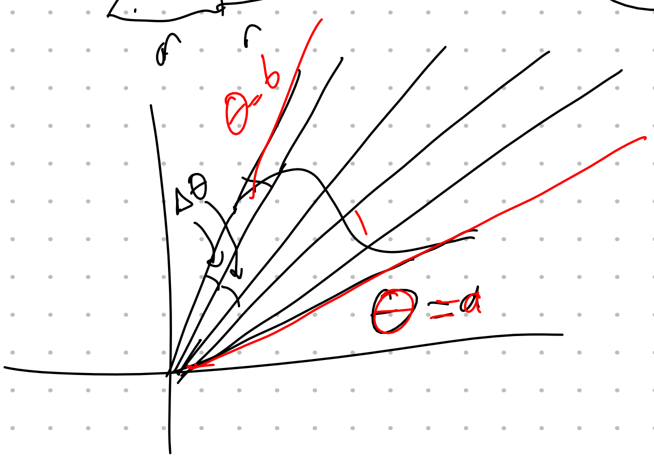
Area $\overset{\text{proportional}}{\sim} \theta$
Area $\sim r^2$ } Area $\sim \theta r^2 \to \frac{2\pi r^2}{2}$

$\boxed{\text{Area} = \frac{1}{2}\theta r^2}$





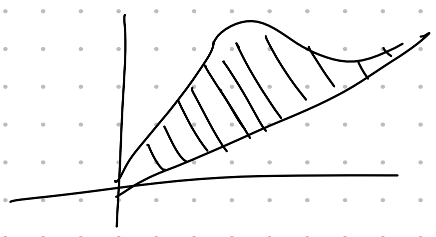
Area $= \pi r^2$

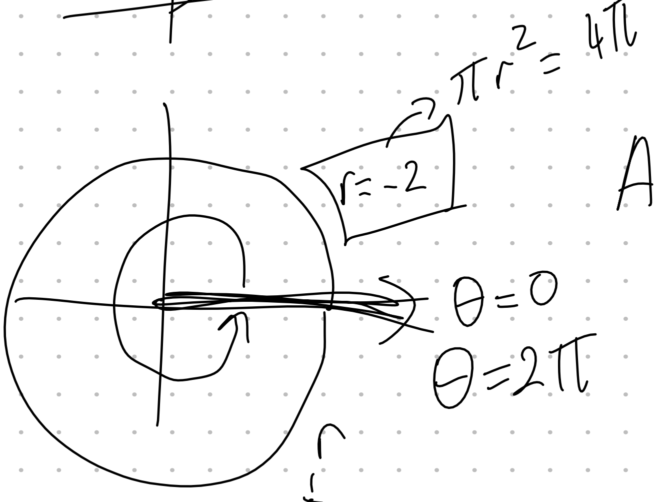


Area of a single wedge $= \frac{1}{2} r^2 \Delta\theta$

Total Area $\approx \sum \frac{1}{2} r^2 \Delta\theta$

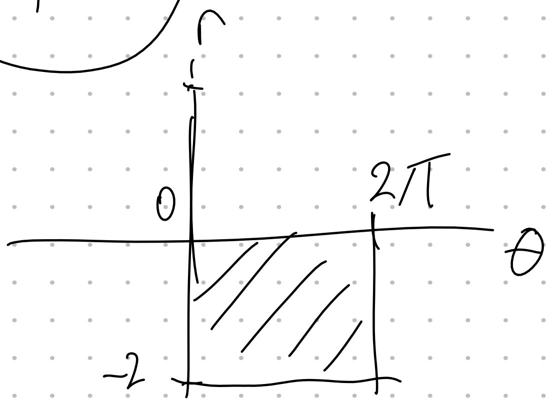
$$\frac{1}{2}\int_{\theta=a}^{\theta=b} r^2 \, d\theta$$

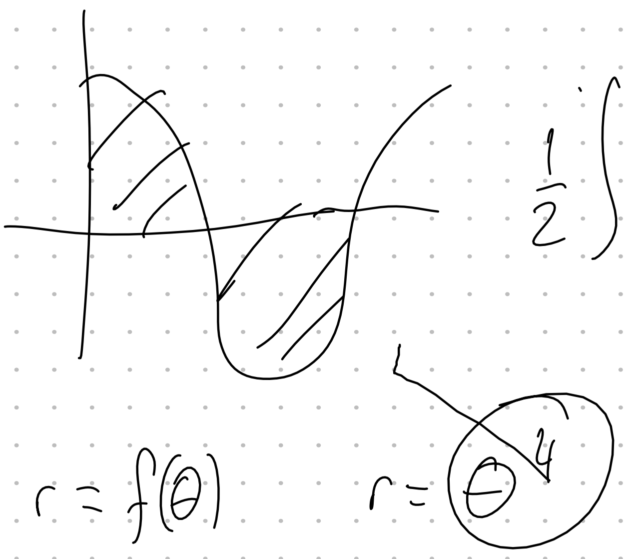




$$\text{Area} = \frac{1}{2}\int_0^{2\pi} \underset{\substack{\parallel \\ 4}}{(-2)^2} \, d\theta = 2\int_0^{2\pi} d\theta$$

$$= 2 \cdot \theta \Big|_0^{2\pi} = \underline{4\pi}$$





$\frac{1}{2}\int r^2 d\theta$ positive

$$\frac{1}{2}\int_a^b r^2 \, d\theta \neq \int_a^b |r| \, d\theta \qquad r = f(\theta) \qquad r = \theta^4$$

$$\frac{1}{2}\int_a^b \theta^8 \, d\theta = \frac{1}{2}\frac{\theta^9}{9}\Bigg|_{a=0}^{b=\pi} \qquad \int_a^b \theta^4 \, d\theta = \frac{\theta^5}{5}\Bigg|_{a=0}^{b=\pi}$$