Example Solve the equation $e^{5-3x} = 10$

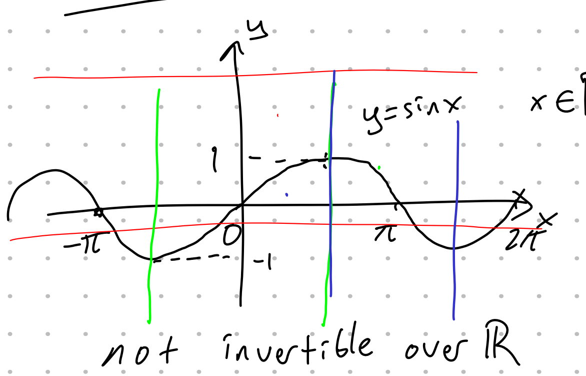
$$5-3x = \ln(e^{5-3x}) = \ln(10)$$
$$-3x = \ln(10) - 5$$
$$x = -\frac{1}{3}(\ln(10)-5)$$

$\boxed{\ln(e^x) = x}$

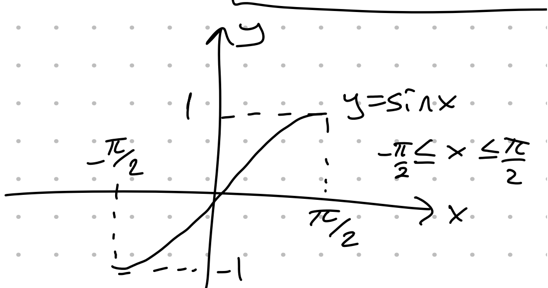
$\ln(e^5) = 5$

$\boxed{\begin{array}{l} f^{-1}(f(x)) = x \\ f(f^{-1}(x)) = x \end{array}}$

Inverse Trigonometric Functions



not invertible over ℝ



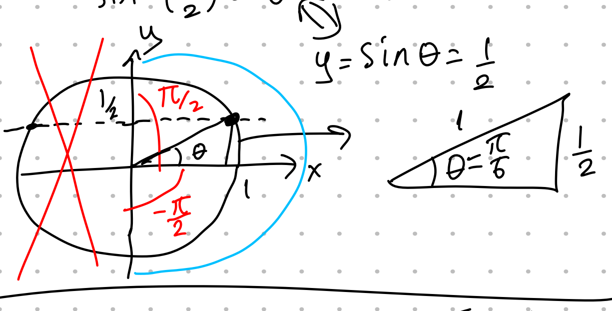
invertible over $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$\sin^{-1}x = y \overset{\text{iff}}{\Longleftrightarrow} \sin y = x$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$\arcsin(x) = \sin^{-1}(x)$

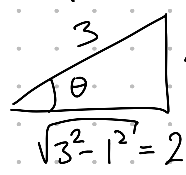
Example Evaluate $\sin^{-1}(\frac{1}{2})$ and $\tan(\arcsin(\frac{1}{3}))$

$\sin^{-1}(\frac{1}{2}) = \theta = \pi/6$



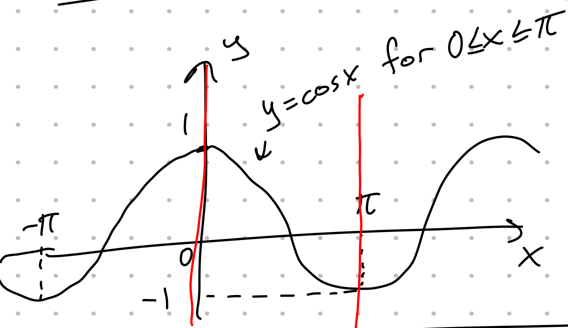
$\arcsin(\frac{1}{3}) = \theta \quad \sin\theta = \frac{1}{3}$

θ has to be in the I quadrant.



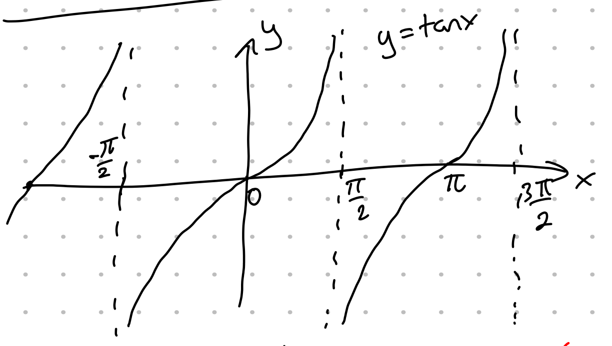
$\sqrt{3^2-1^2} = 2\sqrt{2}$

$\tan(\arcsin(\frac{1}{3})) = \tan(\theta) = \frac{1}{2\sqrt{2}}$



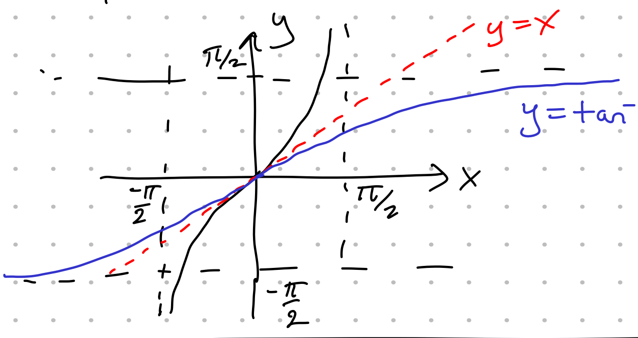
$\cos^{-1}x = y \Leftrightarrow \cos(y) = x$ and $0 \le y \le \pi$

$(\arccos(x) = \cos^{-1}(x))$



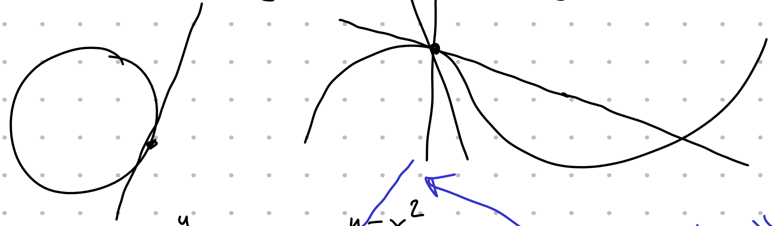
so we can invert $\tan x$ if $-\frac{\pi}{2} < x < \frac{\pi}{2}$

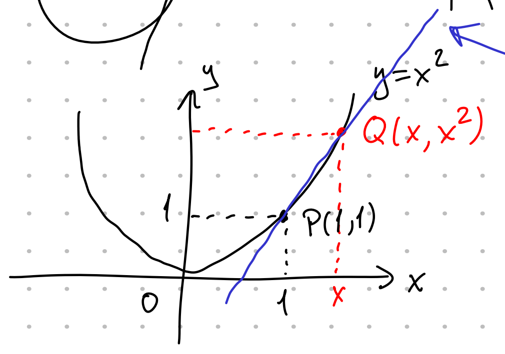
$\tan^{-1}x = y \Leftrightarrow \tan y = x$ and $-\frac{\pi}{2} < y < \frac{\pi}{2}$



so the domain of $\arctan = \tan^{-1}$ is all of ℝ.

2.1 The tangent and velocity problems





"secant line"

let m_{PQ} be the slope of this line.

$$m_{PQ} = \frac{\Delta y}{\Delta x} = \frac{x^2-1}{x-1}$$

x	0	0.5	0.9	0.99	1	1.01	1.1	1.5	2
m_{PQ}	1	1.5	1.9	1.99	X	2.01	2.1	2.5	3

So actually as x approaches to 1 (from either side) m_{PQ} seems to be approaching to 2.

So we define the slope of the tangent line (passing through P(1,1) to be 2. So the equation of the tangent line is $y - 1 = 2(x-1)$ $(m=2 \quad P(1,1))$

$y = 2x - 1$

The Velocity Problem

$$\text{Average velocity} = \frac{\text{distance traveled}}{\text{time elapsed}}$$

Example Suppose that a ball is dropped from the top of a tower 450m above the ground. Find the velocity of the ball after 5 seconds using Galileo's law $s(t) = 4.9t^2$ where $s(t)$ is the distance fallen by a freely falling body after t seconds.

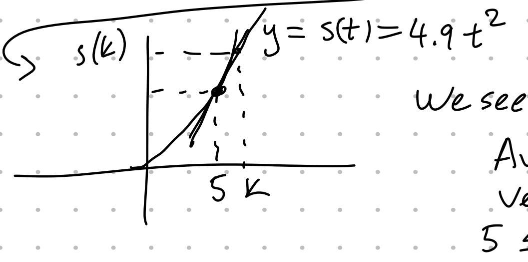
$$\text{Average velocity for } 5 \le t \le 6 = \boxed{\frac{s(6)-s(5)}{6-5}} = \frac{4.9(6^2) - 4.9(5^2)}{1}$$
$$= 53.9$$

Time interval	Average Velocity (m/s)
$5 \le t \le 6$	53.9
$5 \le t \le 5.1$	49.49
$5 \le t \le 5.05$	49.245
$5 \le t \le 5.001$	49.0049

these numbers are approaching to 49 m/s as the interval gets smaller and smaller

So we define Instantaneous Velocity = 49.

$m_{PQ} = \frac{\Delta y}{\Delta x}$ $\quad$ Ave Velocity $= \frac{\Delta \text{position}}{\Delta \text{time}}$



We see that

Average Velocity $5 \le t \le k$ = slope of the secant line between $(5, s(5)), (k, s(k))$