(1 point) Library/Rochester/setAlgebra01RealNumbers/ur_ab_8_3_mo.pg

Let S be $(-\infty, 7] \cup [9, \infty)$.

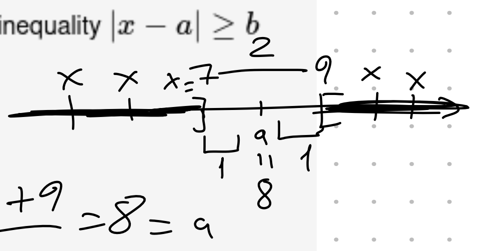
Then S can also be described in set notation by the inequality $|x-a| \geq b$

for

$a =$

and

$b =$



$\frac{7+9}{2} = 8 = a$

$\frac{9-7}{2} = 1 = b$

$|x-a| \geq b$

iff $x-a \geq b$ or $-(x-a) \geq b$

$x \geq a+b$ or $x-a \leq -b$

$a+b = 9$, $x \leq a-b = 7$

$a+b=9$
$+ \quad a-b=7$

$2a = 16 \quad a = 8 \Rightarrow b = 1$

---

Recall $(f \circ g)(x) = f(g(x)) \neq g(f(x))$

Example Given $F(x) = \cos^2(x+9)$ find functions $f$, $g$, and $h$ such that $F = f \circ g \circ h$

$h(x) = x+9 \qquad F(x) = \cos^2(x+9) = [\cos(x+9)]^2$

$g(x) = \cos(x)$

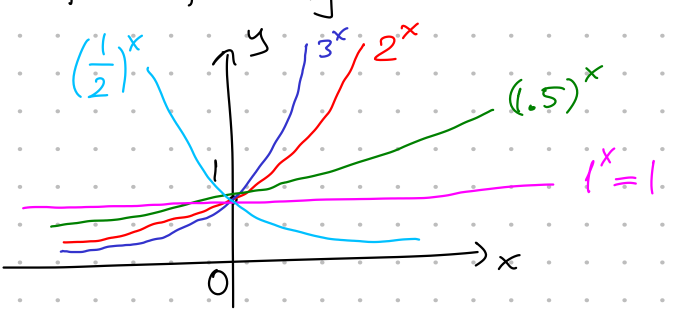
$f(x) = x^2 \qquad F(x) = f(g(h(x))) = (f \circ g \circ h)(x)$

1.4 Exponential functions

$f(x) = b^x$ where $b$ is a positive constant. ($x$: the exponent, $b$: base)

$b^n = \underbrace{b \cdot b \cdot b \cdots b}_{n\text{-times}}$ if the exponent is an integer

$f(x)$ is defined for any real number $x$




domain of $b^x$ is $\mathbb{R}$

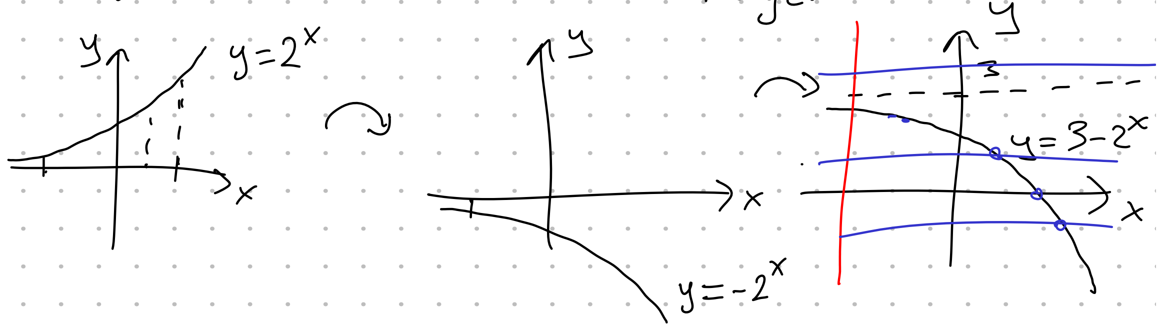
range is $(0, \infty)$

Laws of Exponents

① $b^{x+y} = b^x \cdot b^y$ ② $b^{x-y} = \frac{b^x}{b^y}$

③ $(b^x)^y = b^{xy}$ ④ $(ab)^x = a^x b^x$

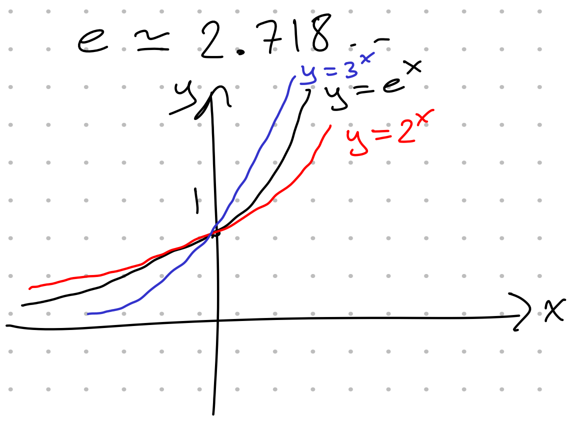
Example Sketch the graph of the function $y = 3 - 2^x$ and determine its domain and range.

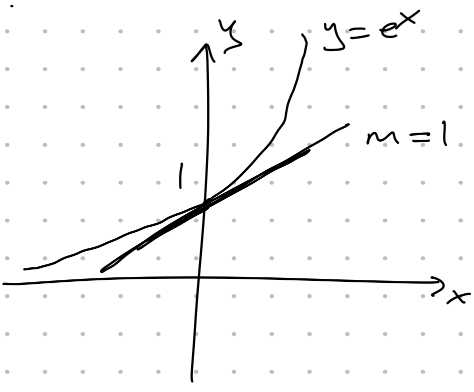



Domain: $\mathbb{R}$ Range: $(-\infty, 3)$

The number $e$

$e \simeq 2.718$



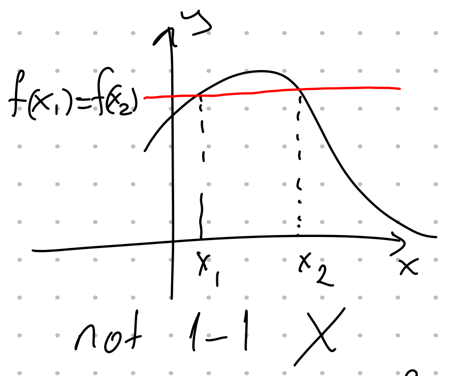




$f(x) = e^x$ : the natural exponential function

1.5 Inverse Functions and Logarithms

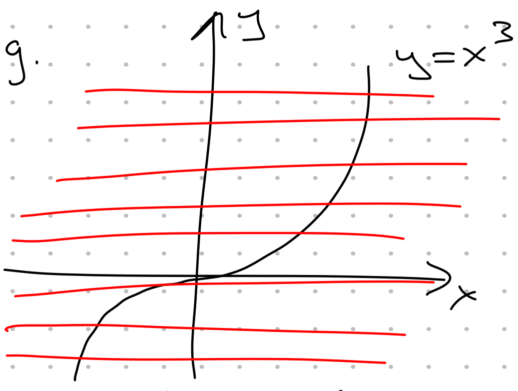
Def$^n$ A function $f$ is called a one-to-one (1-1) function if it never takes on the same value twice; that is $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$

Horizontal Line Test A function is 1-1 if and only if no horizontal line intersects its graph more than once

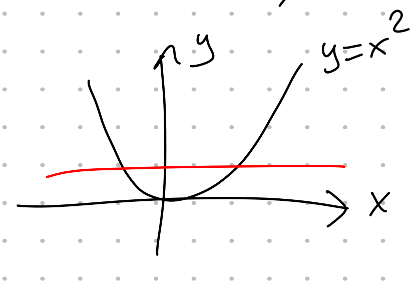



not 1-1 ✗

e.g.




1-1 ✓




not 1-1 ✗

**Def$^n$** Let $f$ be a 1-1 function with domain $A$ and range $B$. Then its inverse function $f^{-1}$ has domain $B$ and range $A$ and is defined by

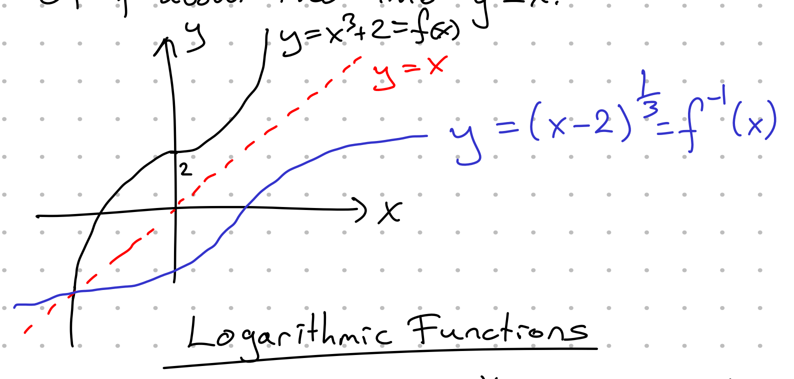
$$f^{-1}(y) = x \quad \text{iff} \quad f(x) = y$$

for any $y$ in $B$.

**Example** Find the inverse function of $f(x) = x^3+2$

$$y = x^3+2$$
$$y-2 = x^3$$
$$\sqrt[3]{y-2} = (y-2)^{\frac{1}{3}} = x$$
$$(x-2)^{\frac{1}{3}} = y \qquad f^{-1}(x) = (x-2)^{\frac{1}{3}}$$

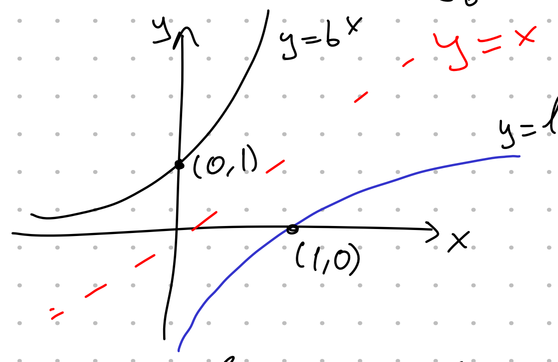
The graph of $f^{-1}$ is obtained by reflecting the graph of $f$ about the line $y=x$.




## Logarithmic Functions

Recall that $y=b^x$ passed the horizontal line test. So it has an inverse: $f(x) = b^x$

then $f^{-1}(x) = \log_b x$




$b=e$ is special

$y = \log_e x = \ln x$

Since $\ln(x)$ is the inverse of $e^x$,

$$\ln(y) = x \quad \text{iff} \quad e^x = y$$
$$\ln(1) = x = 0 \quad \text{iff} \quad e^{x \leftarrow 0} = 1$$
$$\ln(e) = x = 1 \quad \text{iff} \quad e^{x \leftarrow 1} = e$$
$$\ln(e^2) = x = 2 \quad \text{iff} \quad e^x = e^2$$

so, in general,
$$\ln(e^k) = k \quad \text{for any } k \in \mathbb{R}$$
$$e^{\ln(A)} = A \quad \text{for any } A > 0$$

**Example**

find $x$ if $\ln x = 5 \overset{\text{exponentiation}}{\underset{\text{take } \ln}{\Longleftrightarrow}} e^5 = x$

$x = e^5$

$$x = e^{\ln x} = e^5$$