Example Solve $|3x+2| \geq 4$

if and only if (iff) ($\iff$) $3x+2 \geq 4$ or $-(3x+2) \geq 4$

$3x+2 \geq 4$ or $-3x-2 \geq 4$

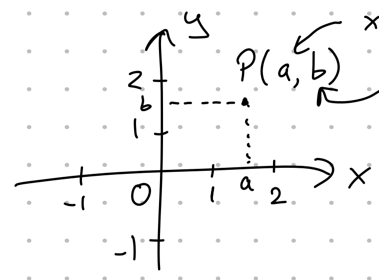
$3x \geq 2$ $\qquad$ $-3x \geq 6$

$x \geq \frac{2}{3}$ $\qquad$ $x \leq -2$

$x \in [\frac{2}{3}, \infty)$ or $x \in (-\infty, -2]$

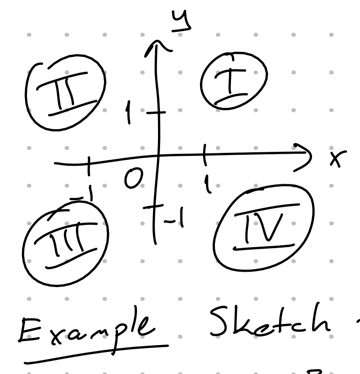
$x \in (-\infty, -2] \cup [\frac{2}{3}, \infty)$

B. Coordinate Geometry and Lines



Points $\iff$ pairs of real numbers

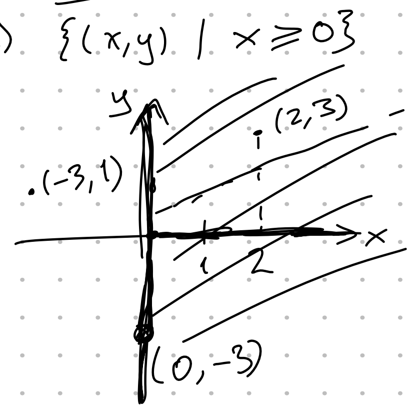
rectangular coordinate system or Cartesian coordinate system, it is denoted by $\mathbb{R}^2$

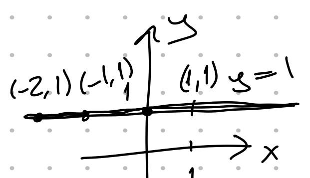


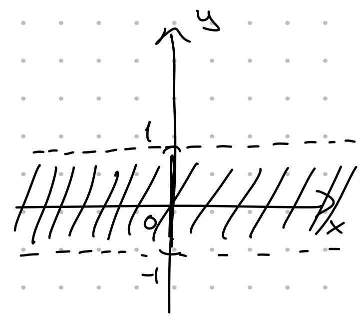
quadrants.

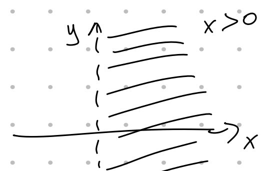
Example Sketch the regions given by the following sets

a) $\{(x,y) \mid x \geq 0\}$ $\qquad$ b) $\{(x,y) \mid y = 1\}$ $\qquad$ c) $\{(x,y) \mid |y| < 1\}$





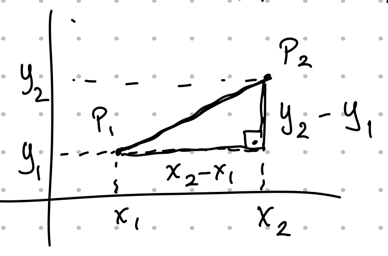




Distance Formula in $\mathbb{R}^2$

Say $P_1(x_1,y_1),\ P_2(x_2,y_2) \in \mathbb{R}^2$

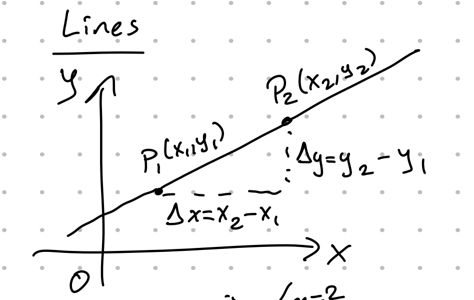
$|P_1P_2|$ = distance between P_1 and P_2 $= \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$



Example The distance between $P_1(1,-2)$ and $P_2(5,3)$ is

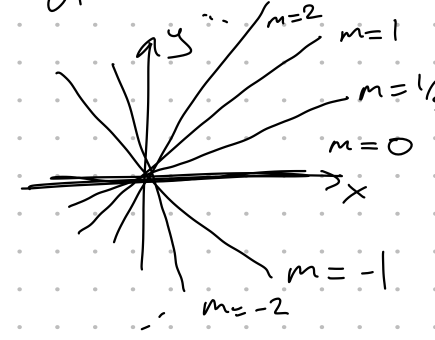
$|P_1P_2| = \sqrt{(5-1)^2 + (3-(-2))^2} = \sqrt{4^2+5^2}$
$= \sqrt{41}$

Lines

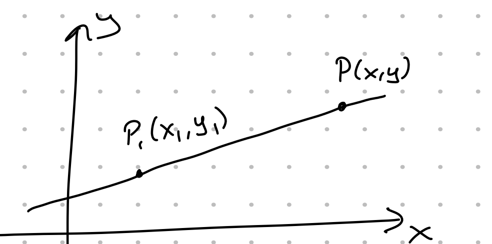


Δx = "change in" x

the slope $m = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x} = \frac{y_2-y_1}{x_2-x_1}$



the slope of a vertical line is not defined!



$y - y_1 = m(x - x_1)$ $\qquad$ $m = \frac{y-y_1}{x-x_1}$

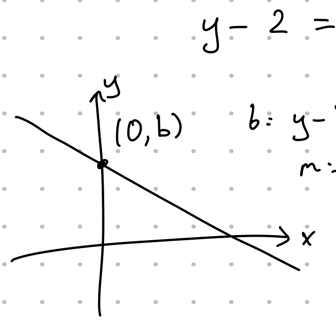
"point-slope form of the equation of a line"

Example Find an equation of the line through the points $(-1,2)$ and $(3,-4)$

$P_1(x_1,y_1)$ $\quad$ P_2 $\qquad$ $m = \frac{-4-2}{3-(-1)} = \frac{-6}{4} = -\frac{3}{2}$

So, using P_1 and m we get

$y - 2 = -\frac{3}{2}(x+1)$



b: y-intercept
m: slope

$x_1 = 0,\ y_1 = b$

$y - y_1 = m(x - x_1)$

$y - b = m(x-0) = mx$

$y = mx + b$

In general every line satisfies an equation in the form

(*) $Ax + By + C = 0$

e.g. $-mx + y - b = 0$ $\qquad$ $(A = -m, B = 1, C = -b)$

vertical lines are of the form $x = c_0$ (some constant)

$x - c_0 = 0$ $\qquad$ $(A = 1, B = 0, C = -c_0)$

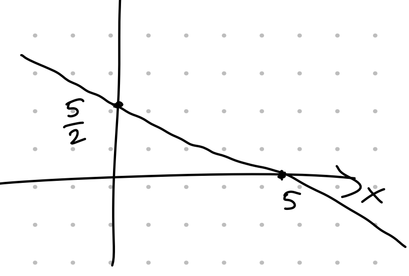
(*) is called a linear equation

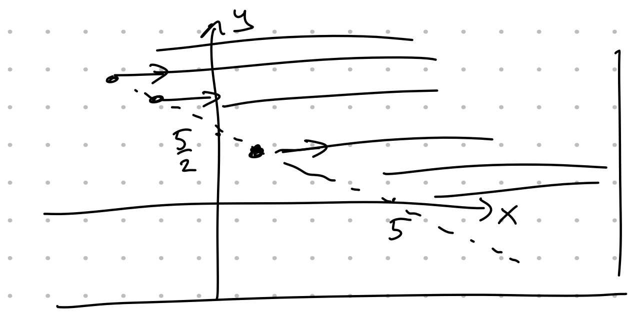
Example Graph $x + 2y > 5$

(first draw $x + 2y = 5$)

Set $x=0$, $2y = 5$ $y = 5/2$ $\quad (0, \frac{5}{2})$

Set $y=0$, $x = 5$ $\quad (5,0)$





Parallel and Perpendicular lines

Say L_1 and L_2 are (non-vertical) lines with slopes m_1 and m_2.

1) L_1 and L_2 are parallel iff $m_1 = m_2$
2) L_1 and L_2 are perpendicular iff $m_1 \cdot m_2 = -1$

Example Find an equation of the line through the point (5, 2) that is a) parallel b) perpendicular to the line $4x+6y+5=0$

$6y = -4x - 5$ $\quad y = \underbrace{\left(-\frac{4}{6}\right)}^{m} x - \frac{5}{6}$ $\quad m = -\frac{4}{6} = -\frac{2}{3}$

(a) $y - 2 = -\frac{2}{3}(x-5)$

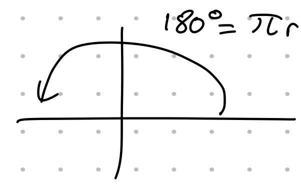
(b) $m_1 m_2 = -1$ $\quad m_2 = \frac{-1}{m_1} = +\frac{1}{\left(+\frac{2}{3}\right)} = 3/2$

$y - 2 = \frac{3}{2}(x-5)$

D. Trigonometry

Angles

$\pi \text{ rad} = 180°$

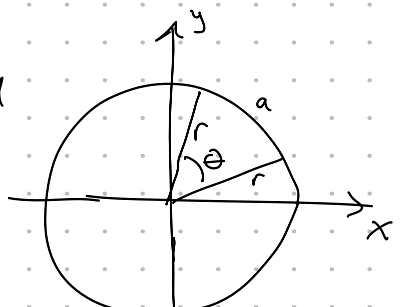


$1 \text{ rad} = \frac{\pi}{\pi} \text{rad} = \frac{180°}{\pi} \approx 57.3°$

$1° = \frac{180°}{180} = \frac{\pi}{180} \text{rad}$

e.g. $60° = 60 \cdot \frac{\pi}{180} \text{ rad} = \frac{\pi}{3} \text{ rad}$

$\frac{5\pi}{4} \text{ rad} = \frac{5\pi}{4} \frac{180°}{\pi} = 225°$



$a = r\theta$ (θ measured in radians)

The standard position of an angle θ