

Homework 3

Üstün Yıldırım

October 2, 2019

These problems are due on Oct 9, Wednesday at the beginning of the lecture.
Justify all your answers.

- Is there a simple, closed curve in the plane with length equal to 12 feet and bounding an area of 12 square feet?
 - What about a simple, closed, regular, (at least) C^1 -curve of length 8π feet and area of 4π square feet? Justify its existence! (You don't have to write an explicit formula. However, you have to show that it exists!) (Piecewise C^1 curves will not get full credit.) (Extra credit: +5 points for showing existence of a simple, closed, regular, C^∞ -curve satisfying $l = 8\pi$ and $A = 4\pi$.)
- Let T denote the torus defined by

$$T = \left\{ (x, y, z) \in \mathbb{R}^3 \mid \left(\sqrt{x^2 + y^2} - a \right)^2 + z^2 = r^2 \right\}$$

for $r < a$. Given that T is a regular surface, show

$$X(u, v) = ((r \cos u + a) \cos v, (r \cos u + a) \sin v, r \sin u)$$

(for $0 < u < 2\pi, 0 < v < 2\pi$) is a parametrization for T . Hint: use Proposition 4 in Section 2.2.

- Find a parametrization for the hyperboloid of one sheet

$$\{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 - z^2 = 1\}.$$

- Find a parametrization for the hyperboloid of two sheets

$$\{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 - z^2 = -1\}.$$

- Section 2.2 Exercise 7
- Section 2.2 Exercise 15